PRACTICAL PHYSICS

AN INTRODUCTORY HANDBOOK
FOR THE PHYSICAL LABORATORY

BY

W. F. BARRETT, F.R.S.E., M.R.I.A., ETC.

PROFESSOR OF EXPERIMENTAL PHYSICS
IN THE ROYAL COLLEGE OF SCIENCE FOR IRELAND

AND

W. BROWN

DEMONSTRATOR OF PHYSICS IN THE SAME COLLEGE

PART I

PHYSICAL PROCESSES AND MEASUREMENTS
THE PROPERTIES OF MATTER

London

PERCIVAL AND CO.

1892

Practical Physics. An Introductory Handbook for the Physical Laboratory. In Three Parts.

Part I.—Physical Processes and Measurements—The Properties of Matter.

Part II.—Heat, Sound, and Light.

Part III.—Electricity and Magnetism—Electrical Measurements.

LONDON: PERCIVAL AND CO.

PREFACE

It is needless at the present day to insist on the fact that the study of Experimental Physics is incomplete without a course of laboratory practice; Professor Huxley wrote upwards of thirty years ago "Mere book-learning in physical science is a sham and a delusion, real knowledge in science means *personal* acquaintance with the facts, be they few or many." Moreover, as a means of education the value of Practical Physics is obvious, for it affords a training in careful observation, reasoning power, and alertness of mind, probably unsurpassed by other kinds of laboratory work; like all practical knowledge, it is a valuable aid in the cultivation of *common sense,* by which is meant a just estimate of the relative value and due proportion of different events.

Nevertheless, it is only in recent years that Practical Physics has been recognised as an integral part of scientific education;* this, doubtless, has been due to various causes: the equipment of a physical laboratory is expensive, but, on the other hand, its working expenses are less than many other kinds of laboratory work; the chief cause,

* The enlightened policy of the Director of the Science and Art Department enabled certificated science teachers, so long ago as 1871, to obtain practical instruction in Physics, Chemistry, and Biology, in the laboratories at South Kensington. At that time systematic instruction in Practical Physics was so much the exception that no English text-book on this subject existed to aid those of us who then took part in giving the course.

however, of the recognition and the growing importance of Practical Physics is doubtless the rapid progress which the last decade has witnessed in electrical discovery and invention, and the consequent development of new and important industries. The number of existing text-books in Practical Physics is still, however, very small compared with the many admirable handbooks on Practical Chemistry. Moreover, less uniformity exists in the mode of teaching the former than of the latter; each laboratory has its own methods, this, whilst not undesirable, throws greater labour on the teacher, who usually prepares for his students MS. notes of each experiment with such brief instructions as are necessary. Though there is a freedom and flexibility in this system of descriptive sheets of separate experiments, it has the serious disadvantage of piecemeal instruction: the student does not see beyond the experiment immediately before him, and has only his laboratory note-book to fall back upon for reference.

The present little volume was undertaken to meet the needs of students in my own laboratory, but it is hoped that it may be found useful to others who are beginning the study of Practical Physics, in a fairly furnished physical laboratory. Whilst the student is supposed to have the use of ordinary instruments of precision, he should be encouraged as far as possible to make for himself the simpler forms of apparatus. The time devoted to Practical Physics by the general student is usually insufficient to allow of this being done, or of much practice in manipulation beyond what is afforded by a systematic course of physical measurement. This is a defect the teacher should recognise: occasional exercises, such as Experiments 7, 18, etc., are therefore introduced to give facility in the construction of physical apparatus, though it

is impossible to include within the limits of this small volume much information in this direction. The arrangement and scope of this book will be seen to differ in some respects from those of the two or three excellent text-books which now exist in England, and the value of which I desire cordially to acknowledge.

Only General Physics is treated in the present Part, but under this head, as will be seen by the Table of Contents, a somewhat wide range of subjects is covered, for, as in other colleges, our course on Practical Physics requires to be adapted to various classes of students. It may therefore be convenient to point out that Chapters I. to V. are intended for general students; Chapters VI. to IX. are adapted chiefly for those who intend to take up Engineering, whilst Chapter X. is intended for students who are taking up more fully Physics or Chemistry. In the Introductory Chapter the main object aimed at is to awaken thought in the mind of the student: if any object to certain parts of this chapter as going beyond the proper boundary of Physics, they can instruct their class to begin at once with the practical work of Chapter II. As far as possible a logical sequence is followed in the order of subjects, and the experiments lead up from simpler to more difficult ones. After stating the nature of each experiment and the instruments needed to perform it, the student is told as briefly as possible what to do, and an illustrative example is worked out for his guidance; a corresponding exercise is then given for the student to perform. The object of each lesson is to confine the student's attention to the particular experimental work in hand, and to avoid distracting his attention and encumbering the page with theoretical considerations.

The student who uses this book ought therefore to have already attended a course of lectures on Experi-

mental Physics, and if unacquainted with the theoretical explanation of the experiment he is pursuing, he should consult at his leisure the references given in the footnotes. In some cases, however, where the ordinary text-books give less help in this direction, my collaborateur has added in the form of appendices the necessary mathematical demonstrations. As the labour of the preparation of this volume, with the exception of Chapters I. and X., has been shared by my colleague, Mr. W. Brown, his name is included on the title-page.

In conclusion, I have gratefully to acknowledge the kind help given by many friends in the revision of the proof-sheets; at the same time it will of course be understood that those who have been good enough to add to the accuracy of this little volume are in no wise responsible for its shortcomings. To Principal Garnett, M.A., D.C.L., and to Professor Fitzgerald, F.T.C.D., F.R.S., thanks are especially due for their valuable revisions and suggestions, and to Mr. F. J. Trouton, M.A., D.Sc., for his careful reading of the whole of the proofs. I have also warmly to thank Professor Rücker, M.A., F.R.S., and the Rev. M. H. Close, M.A., Treasurer of the Royal Irish Academy, for their kind and important revision of the parts submitted to them. For several of the woodcuts I am indebted to the kindness of Mr. Hicks of Hatton Garden, London, and Mr. Yeates of Dublin; these or other well-known instrument-makers can supply all the instruments required in this book.

W. F. BARRETT.

ROYAL COLLEGE OF SCIENCE FOR IRELAND,
September 1892.

CONTENTS

CHAPTER I

INTRODUCTION AND DEFINITIONS

CHAPTER II

MEASUREMENT OF SPACE

Section I.—*Length Measurement*

Section II.—*Angular Measurement*

Section III.—*Area Measurement*

Section IV.—*Volume Measurement*

CHAPTER III

MEASUREMENT OF TIME

CHAPTER IV

MEASUREMENT OF MASS

CHAPTER V

MEASUREMENT OF FLUID PRESSURE

CHAPTER VI

MEASUREMENT OF FORCE

CHAPTER VII

MECHANICAL PROPERTIES OF MATTER—SOLIDS

CHAPTER VIII

MECHANICAL PROPERTIES OF MATTER—SOLIDS (*Continued*)

CHAPTER IX

MECHANICAL PROPERTIES OF LIQUIDS

CHAPTER X

MOLECULAR PROPERTIES OF FLUIDS

APPENDIX

TABLES

ERRATA.

In Fig. 7, page 18, the letters A, B, C have been omitted from the three outer feet, and the letter O from the central foot, shown in the woodcut.

On page 19, line 3 from bottom of page, *for* "D" *read* "O"; and in the next line, *for* the letters "E" and "F" *read* "S" and "D."

On page 20, line 5 from top, *for* "used" *read* "measured."

On page 235, line 3 from bottom of page, *for* $\sqrt{2\frac{o}{c}}$ *read* $\frac{o}{e}\sqrt{2}$.

fore, called elements. Matter is made up of molecules,*

* A molecule is defined by chemists as being the smallest portion of a substance that is capable of existing by itself, the smallest cluster of atoms

TABLES

CHAPTER I

INTRODUCTION AND DEFINITIONS

In the physical universe, as it is apprehended by us, two classes of things exist independently of our senses: these are matter and energy. Both these things are unalterable in quantity by any process known to man, that is to say they are uncreatable and indestructible by any human power. Matter and energy are invariably associated, neither is known to us nor perceivable by us apart from the other. Matter is that which occupies space, is the vehicle of energy and possesses the common attribute of inertia. Energy is the power of doing work, and depends upou the position or motion of matter.

The ultimate nature of matter is unknown; ordinary or gross matter, as distinguished from the matter of the ether, can be broken up by chemical and physical processes to about seventy different kinds of substances; up to this time all of these have resisted further decomposition, they are, therefore, called elements. Matter is made up of molecules,*

* A molecule is defined by chemists as being the smallest portion of a substance that is capable of existing by itself, the smallest cluster of atoms

and molecules of immutable atoms; an atom of hydrogen, for example, keeps unchanged for ever, in spite of all its combinations and dissociations, its own qualitative and quantitative properties; furthermore, its period of vibration is the same, whether it be on the earth, or in Sirius, or in the Sun.

There is no growth nor decay, no generation nor destruction among the atoms. Hence no process of evolution can be going on among them, for evolution implies continuous change. In fine, atoms present the appearance, as the late Sir John Herschel and Professor Clerk Maxwell have remarked, of being "manufactured articles," * and this precludes the idea of their being eternal and self-existent. Energy, on the other hand, is mutable, a definite portion cannot be identified, but is perpetually undergoing transformation, as from the mechanical motion of a mass to the molecular motion of heat, or passing into the disturbance of the ether, known as radiation, and thus entering or escaping from the earth, but withal there is no loss nor destruction. The sum of all the energy in the universe, like the sum of all the matter, is a constant

to which can be attributed all the chemical properties of the whole substance, hence it is the unit which the chemical formula of the substance ought to represent. Atoms are the indivisible constituents of a molecule.

* On account of their uniformity, every individual atom of the vast multitudes that make up each group or element being exactly alike, "as though they had all been cast in the same mould like bullets," and no gradual transition existing between the atoms in one group and those in another. As the atoms are, we believe, "the only material things which still remain in the precise condition in which they first began to exist," unalterable by any natural processes, we are led to conclude that it is not to the operation of these processes their uniformity is due. See Clerk Maxwell's *Heat*, p. 340, and Art. "Atoms" in *Ency. Brit.*

quantity: this is the doctrine of the conservation of energy. There are two broad divisions of energy—namely (1) the energy due to the relative motion of matter, called *kinetic energy*, and (2) the energy due to the relative position of matter, which is called *potential energy*. These are constantly interchanging, a typical example being seen in the swing of a pendulum. The different forms of energy are not equally available for useful, *i.e.* directed, work; the ratio of the amount of energy which can be transformed into work to the whole energy possessed by a system is the *availability* of any given form of energy.

Physics, in its widest sense, is the study of the dynamics of matter and energy. Practical physics is the personal investigation with measurement of the mechanical laws of matter and energy. For this purpose instrumental appliances are necessary, and for accurate measurement instruments of precision are requisite. Acquaintance with these instruments and a knowledge of their use form, therefore, an integral part of our study. But in measurement we may either find the proportion one thing bears to another of the same kind, as the depth of one river to another, or we may find the magnitude of the thing itself referred to some particular unit, as a river 20 *feet* deep. In the former case the ratio found is a number, and is a complete expression which everywhere holds good. In the latter case two factors are involved—namely, a number and a unit, which latter must be a quantity of the same kind as the measurement we wish to make; in the above case 20 is the number and a foot the unit. This expression is a statement of a

physical quantity. Hence the measurement of a physical quantity resolves itself into the selection of a suitable unit, and the determination of the numbers of this unit, or fractional parts of a number, which are contained in the quantity measured. Experiment alone can determine this value. It will, therefore, be seen that the object of all physical measurements is to ascertain either a simple numerical ratio, or the number of units in the particular thing under experiment. Thus the determination of the specific gravity of a body is the discovery of the ratio of the weight of that body to the weight of an equal bulk of water; the resulting ratio is a mere number, true everywhere. The determination, on the other hand, of the wave length of a particular colour is the discovery of the number of units, or fractions of a unit, of length which are contained in the coloured ray. To render this latter measurement capable of verification anywhere, we must be able to define our unit accurately, and also we must be certain that the unit itself does not alter in time, or in different places on the earth's surface. The information given by such a measurement, though the experiment is, as a rule, more troublesome, is obviously more complete than the information given by the measurement of a ratio.

As physics includes the phenomena of gravitation, heat, light, sound, electricity, magnetism, etc., a very large number of diverse physical quantities have to be dealt with, and accordingly a number of different units are necessary. But these different units can be reduced, ultimately, to three *fundamental units.** The fundamental

* Temperature, however, is not yet included in any authorised way.

units chosen, which are also *fundamental ideas,* are a unit of *space* or *length,* a unit of *time,* and a unit of *mass.* Time and space, it is true, cannot be considered *things,* like matter and energy, yet they are to us a necessary relationship of things, and therefore they both enter into all our knowledge, and even conception, of phenomena.

The idea of *space* is derived from the recognition that every place or substance has a definite position with respect to every other place or substance, the distance between one object and another being independent of any material thing between them. As we cannot describe the place of a body without reference to some other body, our knowledge of space is relative; the absolute position of any point we cannot determine. Space, as we know it, has three directions or dimensions at right angles to each other, and entirely independent of one another; thus in the upper or lower corner of a room there are three diverging lines—the two horizontal ones indicate two of these directions and the vertical line the third; every other direction is compounded of these. Hence the position of any point in space can be defined by reference to these three directions, and any point in space can be reached by moving in the direction, or a combination of the directions, of these three lines. As matter occupies space all material substances possess three dimensions, which we commonly indicate by speaking of the length, breadth, and depth of a body.*

* We may conceive of space of one dimension, such as a geometrical line, either straight or curved, where there is only length; or of space of two dimensions, such as a plane or curved surface, where there would be only length and breadth; but the smallest particle of matter, as we know

The idea of *time* is derived from the recognition of successive states of consciousness. Of absolute time or duration we know nothing, we cannot describe the time of any event without reference to some other event. Hence our notion and knowledge of time, like that of space, is purely relative. A measure of time may be obtained from the statement in the first law of motion, where Newton defines that a mass left to itself moves uniformly, that is, equal times are taken to traverse equal spaces. The common measurement of time depends on the rotation of the earth about its axis; this period is assumed invariable, but the apparent solar day is not so, and therefore the average length of a solar day is used and called the *mean solar day*. Even here we probably do not possess a perfectly uniform and unchangeable standard; whether such an ultimate standard of time can be found, as has been suggested, in the period of vibration of the molecules of a hot gas under definite conditions, we need not consider. It is sufficient to know that the astronomical measure of time, though not absolutely constant, according to Newton's definition, is practically so for all ordinary purposes.*

In selecting standard units of space, of time, and of mass it is necessary that they be exactly defined, so that

it, possesses three dimensions. There may be regions of space more limited than ours, and beings inhabiting such regions, to whose limited movements and conceptions our threefold space and motion would be inconceivable and miraculous; and there may be regions of ampler space than ours, and beings inhabiting those regions, who would, in like manner, look down upon the limitations of our movement and of our faculties. See Hinton's suggestive little book, *What is the Fourth Dimension?* (Sonnenschein and Company).

* See Thomson and Tait, *Nat. Phil.* vol. i. part 1, p. 460.

they can be accurately reproduced at any place or time. The common *unit of length* in English-speaking countries is the British standard *yard*, which is the distance between two marks on a certain bronze bar in London when at a temperature of 62° F. The *unit of length* in the metric system is the standard *metre*, which is the distance between the ends of a certain platinum bar in Paris at a temperature of 0° C. This length is equivalent to 39·37043 British inches. The *unit of time* is the same everywhere, and is the *second*, which is the $\frac{1}{86400}$ part of the mean solar day. The *unit of mass* is either the British standard *pound*, a certain mass of platinum kept at the Standards Office in London, or in the metric system the *kilogramme*, a certain mass of platinum kept, like the metre, in the French Archives; the kilogramme is equivalent to 2·2046 pounds.

The metric unit of length was originally derived from the ten-millionth of the distance from the Equator to the North Pole, measured along the meridian of Paris, and the kilogramme was derived from the mass of one-thousandth part of a cubic metre (one cubic decimetre) of pure water at 4° C. Decimal multiples indicated by Greek prefixes, and decimal sub-multiples indicated by Latin prefixes, are used in the metric system; thus a kilometre is a thousand metres, a millimetre (mm.) is one-thousandth of a metre, a micromillimetre ($\mu\mu$.) one-millionth of a millimetre, the prefix *mega* meaning one million times (see Table II., where the British equivalents to the metric system are given).

The inconvenience of, and want of correlation in, our British system of weights and measures has led to the

general adoption by scientific men of the metric system. As physical measurements are usually of small quantities, the centimetre and the gramme have been selected as the units of length and mass; accordingly the three fundamental units we shall use in this book, and which are in general use by physicists throughout the world, are the *Centimetre* (cm.), the *Gramme* (grm.), and the *Second* (sec.) The system based on these units is known as the C.G.S. system. Its units are, however, inconveniently small for engineers, who in English-speaking countries generally employ the British system of units, viz. the *Foot, Pound,* and *Second*; we shall therefore occasionally employ these units in those measurements useful for engineering students.

In the Exercises that follow we shall begin with the linear measurement of space, then the measurement of areas and volumes, the measurement of time, the measurement of mass and density, the measurement of force. We shall next enter on the study of the properties of matter, beginning with solids, then liquids and gases, and lastly the molecular properties of fluids will be investigated. The appendices will be found to contain proofs of some of the problems, and a series of tables are given at the end. Each student is expected to provide himself with either Bottomley's or Woodward's handy table of four-figure logarithms, and with a rough notebook for entering his results, which must be afterwards expanded and written out in a carefully-kept notebook.

Definitions.

For our present purpose the following definitions will suffice; others will be given as required later on (see Table I.) Fundamental units are printed thus [L].

LENGTH.—The unit of length is 1 centimetre = [L].

MASS.—The unit of mass is 1 gramme = [M].

TIME.—The unit of time is 1 second = [T].

AREA (A) is the product of length and breadth, or surface; unit area is the area of a square whose side is 1 cm. = [L^2].

VOLUME (V), or capacity, is the product of length, breadth, and depth; unit volume is a cube whose side is 1 cm. = [L^3].

DENSITY (ρ) is the mass or quantity of matter per* unit volume, and is the number of units of mass in unit volume of the substance, $\rho = M/V$; unit density = [M] / [L^3].

VELOCITY (v) is the time rate of change of position; unit velocity is the velocity of 1 cm. per sec. = [L] / [T].

ACCELERATION (a) is the time rate of change of velocity; unit acceleration is the gain of unit velocity (*i.e.* 1 cm. per sec.) every second = [L] / [T^2].

MOMENTUM (m) is the quantity of motion in a moving body; and is the product of the velocity of a body into its mass, $m = Mv$; unit momentum = [M] [L] / [T].

FORCE (F) is the time rate of change of momentum; and is the product of mass into acceleration, $F = Ma$; unit force = [M] [L] / [T^2]. The unit of force is the *dyne*, and is that

* The English equivalent of "per" is "divided by," and is indicated by the sloping line /, or by the negative sign to the index; thus velocity is L/T or LT^{-1}, and so on.

force which, acting on a gramme of matter for 1 second, generates in it a velocity of one centimetre per second. The moment of a force about a point or axis is the product of the magnitude or component of the force in a plane perpendicular to the axis, into the distance from the axis of the line of action of the component. If a force of magnitude F acts at a distance p from and perpendicularly to the given axis, Fp is the magnitude of its moment about the axis.

WORK (W) is done by a force on a body when the body is displaced in the direction of the force, and is measured by the product of the force into the displacement (s), W = Fs. The unit of work is the *erg*, and is the force of a dyne moving its point of application through the length of a centimetre, therefore = $[M]\,[L^2]\,/\,[T^2]$.

ENERGY (E) is the capacity for doing work, and is measured by the quantity of work it can do; it has therefore the same dimensions as work: $[M]\,[L^2]\,/\,[T^2]$. If the energy given out or gained by a system (1) results in motion, the system is said to lose or gain *kinetic energy*; if, on the other hand, (2) the result be a change of configuration in its parts, the system is said to gain or lose *potential energy*.

POWER or activity (P) is the mean rate at which a force does work in a given time, and is measured by the quotient of the work done in the time by the time taken: P = W/T. The unit is one erg per second = $[M]\,[L^2]\,/\,[T^3]$.

CHAPTER II

MEASUREMENT OF SPACE

SECTION I.—*Length Measurement*

Experiment **1.—To find the distance between two marks on a sheet of paper.**

Instruments required.—An ordinary or "hair" compass, a beam compass, and a millimetre scale.

Fig. 1.

By adjusting the points of the compass to the extremities of the length to be measured, and then applying the compass to the scale, the number of units and fractional parts of a unit of length in the given space can be determined. To enable the adjustment of the compasses to be made with accuracy, a fine motion can be given to one leg of the "hair compass" (Fig. 1) by means of a screw, or an eccentric, A.

For lengths greater than the space of an ordinary compass the "beam compass" (Fig. 2) is employed. This consists of a long brass rod A, with two steel points B and C, the latter capable of sliding along the rod A, and of being clamped

in any required position by the screw F. The point B has also a fine adjustment by means of the screw D.

To make a measurement, apply the compasses to the two points whose distance apart is required, adjusting

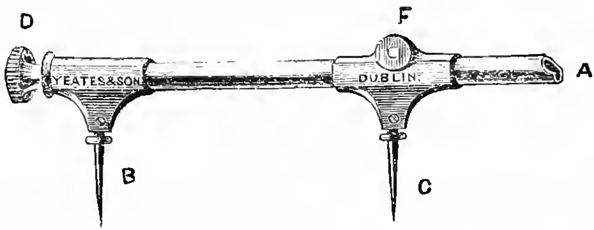


Fig. 2.

accurately by means of the screw D, then apply the points to the scale, estimating by eye the decimal parts of a scale division. Repeat the measurement three or four times, using different parts of the scale. The brass rod A is itself graduated in some instruments, so that the required length can be read directly.

Example.—Find on a millimetre scale the length that corresponds to 5 inches. Enter your results thus:—

1st trial . .	12·7	mm.
2nd „ . . .	12·65	„
3rd „ . . .	12·75	„
Mean =	12·7	mm.

Exercises.

1. Find the distance between two scratches on a plate of glass.

2. Test the accuracy of the ruling of the piece of millimetre or curve paper supplied to you.*

3. Compare three centimetre scales, engraved respectively on glass, steel, and paper.

Experiment 2.—**How to use a vernier and to find the "least count" of a vernier.**

Instruments required.—A scale with its accompanying vernier.

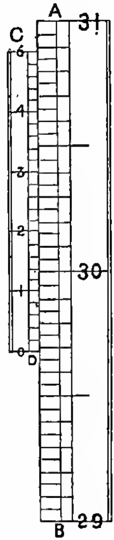


Fig. 3.

In Fig. 3 is shown a scale and vernier wherein 25 divisions on the vernier CD correspond to 24 divisions on the scale AB. Each scale division is equal to $\frac{1}{20}$ or ·05 of an inch, therefore each vernier division will be

$$\frac{24}{25} \times \frac{1}{20} = 0{\cdot}048 \text{ of an inch.}$$

Hence each *scale* division *exceeds* each *vernier* division by $({\cdot}05 - {\cdot}048) = {\cdot}002$ of an inch, which is the "least count" of this particular vernier.

In order to use the vernier after getting the "*least count*" (d), *first read off the graduation* (R) *immediately below the zero of the vernier; then count up the vernier from its zero to the division that exactly coincides with a scale division, say the nth vernier division from its zero; the total reading will then be* ($R + nd$).

* Millimetre paper is paper ruled into small squares, such as is shown in Fig. 19; it is essential for work in the physical laboratory, and can be obtained from Messrs. Williams & Norgate, of London, or Messrs. Dollard,

The general theory of the vernier is given in Appendix, § 1.

Example.—The vernier zero stands between 30·35 and 30·4 inches, and coincidence occurs at the 18th division from zero, therefore the total reading is

$$30{\cdot}35 + (18 \times {\cdot}002) = 30{\cdot}386 \text{ inches.}$$

In practice proceed as follows: (i.) Read first the inches and nearest fraction of an inch on the scale, *e.g.* (Fig. 3) 29·65. (ii.) Then note the point of coincidence between the divisions on the scale and vernier, and read the nearest whole number on the vernier at or below the point of coincidence. Let this be 2; add this to the scale reading in the second place of decimals, *e.g.* 29·67. (iii.) Then observe how many (if any) of the divisions on the vernier are between the point of coincidence and the figure 2; let this be 3, as the least count is $= {\cdot}002$ of an inch, $3 \times {\cdot}002 = {\cdot}006$; add this to the previous reading, then 29·676 is the final reading. This is the usual form of scale and vernier used in standard barometers in the United Kingdom.

Note.—As each figure marked on the vernier includes 5 divisions, there will be 15 vernier divisions at the figure 3; hence $15 \times {\cdot}002 = {\cdot}03$ of an inch, and so on with the other figures on the vernier, which therefore indicate the number of hundredths of an inch to be added

of Dublin, and other stationers. It is to be had ruled into squares whose sides are fractions of an inch or of a centimetre, the most convenient paper being that which has every fifth or tenth division ruled in a thicker or a different-coloured line.

to the reading of the scale. For ordinary barometric purposes this reading is sufficient, and indeed the construction of the barometer in general makes any finer reading only a misleading attempt at accuracy.

Exercise.

Find the "least count" of the verniers attached to the cathetometer (Fig. 8), barometer (Fig. 35), calipers (Fig. 4), spectrometer, and sextant. Read the barometer each day for a week and enter your readings.

Experiment 3.—**To find the length of a cylindrical bar.**

Instrument required.—The slide calipers.

The slide calipers (Fig. 4) consist of a rectangular bar of metal A, with one jaw B fixed at right angles to it at

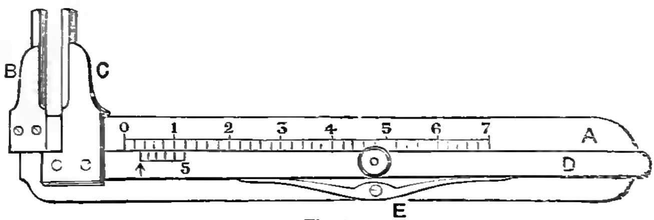


Fig. 4.

one end, and another jaw C fixed to the sliding piece D, which is kept in a half V-groove by means of the spring E. The jaw C moves parallel to B; A has a scale in centimetres engraved upon it, and D has a vernier also engraved on it, which enables the scale to be read to tenths of a millimetre.

To make a measurement, open the jaws and insert the

bar, then close C till the two jaws just touch the ends of the bar; now read off the scale and vernier.

Example.—The scale in another instrument is divided into twentieths of an inch, and the "least count" of the vernier is ·001 inch. The zero of the vernier stands between 1·55 and 1·6 inch, and coincidence occurs at the 14th division from zero. Therefore the length of the bar is

$$1{\cdot}55 + (14 \times {\cdot}001) = 1{\cdot}564 \text{ inches.}$$

Note.—In order to measure an inside length—such, for example, as the internal diameter of a glass tube—the width of the two jaws must be added on to the scale reading. This is called the "outside correction."

Exercises.

1. Find the mean diameter of a bar, and calculate its cross sectional area and volume.

2. Find the mean diameter of a sphere, and calculate its surface and volume.

3. Find the mean thickness of the glass at the mouth of a jar by direct measurement and by taking the *external* and *internal* diameter (see Table III., 6).

Experiment 4.—**To measure the diameter of a wire.**

Instruments required.—The wire-gauge or micrometer screw, and a millimetre scale.

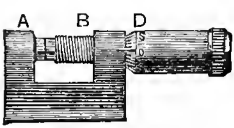


Fig. 5.

The micrometer screw (Fig. 5) consists of a piece of metal turned twice at right angles; a steel plug A with plane end is fixed in one bend or arm, and in the other arm a fine screw B works

smoothly; its face is also plane and parallel to that of A.

Along a fixed arm a scale is engraved, and the cap D, which is a part of the screw B, is also divided into a certain number of equal divisions. One complete turn of the cap moves the faces A and B nearer to or farther from each other by an amount equal to the distance of one scale division of the fixed arm.

To make a measurement, screw back B, insert the wire, screw up B till the wire is just caught between A and B, then read off the position indicated on the fixed arm and the division on the cap.

Example.—(Each division on the fixed arm is $= \frac{1}{2}$ mm., and there are 20 divisions on the cap D, therefore each division on the cap measures to $\frac{1}{20} \times \frac{1}{2} = \frac{1}{40}$ mm.) When A and B are just touching, the reading on D must be taken. It ought to be 0; if not, the difference must be noted and allowed for.*

Let the thickness of a piece of wire give a reading of 3 on the fixed scale and 18 on the cap, therefore the diameter of the wire is $(3 \times \frac{1}{2}) + \frac{18}{40} = 1{\cdot}95$ mm.

Another and superior form of this instrument, first devised by Mr. Yeates of Dublin, is shown in Fig. 6.

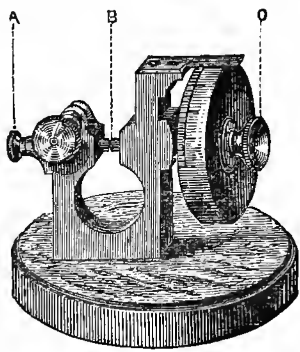


Fig. 6.

* Care must be used that similar pressure, as indicated by the sense of touch, is given in each case.

Here greater accuracy is obtained by enlarging the engraved circle attached to the micrometer screw, which is turned by the milled head C. The zero of the instrument can also be adjusted by loosening the clamping screw which fastens the solid plug A. A scale engraved on D shows the number of complete revolutions made by the screw; if the pitch of the screw be 100 to the inch, and the engraved circle be divided into 100 parts, each division will correspond to $\frac{1}{100}$ of $\frac{1}{100}$ of an inch or $\frac{1}{10000}$ of an inch.

Exercises.

1. Find the diameter of a wire; measure at several points and take the mean.
2. Find the average thickness of a strip of paper.
3. Find the thickness of a piece of microscopic covering glass.

Experiment 5.—**To measure a thin plate and the radius of curvature of a curved surface.**

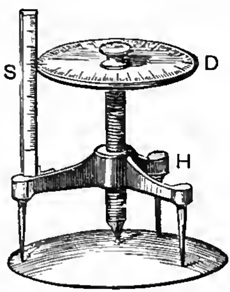


Fig. 7.

Instruments required.—A spherometer, a plane glass surface, and a millimetre scale.

The spherometer (Fig. 7) consists of a triangular frame H supported on three feet, whose extremities form an equilateral triangle, and in the middle of the frame is a fourth foot capable of moving up and down perpendicularly by means of the fine screw chased upon it.

The disc D, which is graduated round the circumference,

turns with the central foot; the edge of this disc serves as an index to read the scale S which is attached to the frame.

One complete turn of the disc D raises or lowers the centre foot through one division of the scale S, and therefore gives the perpendicular distance between the central foot and the plane of the other three feet.

To use the spherometer, first test the zero by placing the instrument on the glass plane and screw D up or down till the whole four feet rest equally on the surface. This is best ascertained by the sense of touch, *i.e.* when the rocking of the instrument on the plane just ceases.

To find the thickness of a thin plate, place the central foot over the thin plate, the other three feet resting on the plane surface; then raise or lower the screw until the four feet bear equally; when the rocking ceases read the distance through which the screw has been raised. The difference between this reading and the zero is equal to the thickness required.

To find the radius of curvature of a surface, such as a lens or mirror, place the whole instrument on the curved body, and raise or lower the central foot until all four bear equally, and no rocking is felt.

If R = the radius of curvature of the surface to be measured,

l = the distance AB,

a = the distance of D above or below the plane of A, B, C, as measured by the scale E and disc F,

then $$R = \frac{l^2}{6a} + \frac{a}{2}$$ (see Appendix, § 2).

Example.—(The scale S is divided into $\frac{1}{4}$ mm. and there are 100 divisions on the disc D, therefore one division on D reads to $\frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$ mm.). The reading on the scale S stands between 4 and 5, and the disc reads 70, a convex lens being used.

$$a = (4 \times \tfrac{1}{4}) + \tfrac{70}{400} = 1{\cdot}175 \text{ mm., and } l = 42 \text{ mm.,}$$

$$\therefore \text{R} = \frac{42^2}{7{\cdot}05} + \frac{1{\cdot}175}{2} = 250{\cdot}8 \text{ mm.}$$

Exercises.

1. Find the thickness of a piece of microscopic covering glass.

2. Determine the radius of curvature of a concave mirror and also that of a convex lens.

Experiment 6.—**To measure the difference in height between two marks by means of the cathetometer.**

Instruments required.—A cathetometer, a centimetre scale, and a plumb line.

The cathetometer (Fig 8) consists of a strong vertical support AB, with a scale engraved upon it.

This rod is capable of moving round a vertical axis by means of a journal at its lower end turning in the tripod stand.

This tripod stand contains three levelling screws, which enable the rod AB to be made perpendicular, as indicated by the two spirit-levels placed at right angles to each

other. This is the first adjustment to be made in the use of the cathetometer.

A carriage C, supporting the telescope T, slides up and down the vertical rod, and can be clamped at any position. This carriage has a vernier attached to it (see enlarged drawing, Fig. 9), which, together with the scale, enables us to read off the vertical height of the telescope.

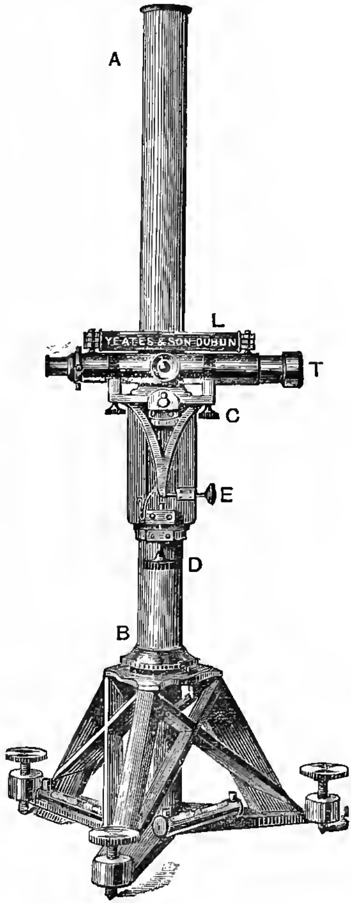


Fig. 8.

D is a screw with fine motion to give a small final adjustment to the telescope when making an observation.

Before taking each observation the telescope is carefully levelled by means of the screw E and the graduated spirit-level L on T. The eye end of the telescope contains fine cross hairs, which must be focused by adjusting the eye-piece. To avoid parallax * the telescope should

* By parallax is meant the displacement of the image of an object due

first be turned to a distant object and the cross hairs focused for parallel rays, then focus the object glass on the mark to be read, by means of the rack and pinion; now carefully adjust the eye-piece till both mark and cross hairs are seen distinctly, and no alteration of their relative positions is made by moving the eye slightly out of the axis of the telescope.

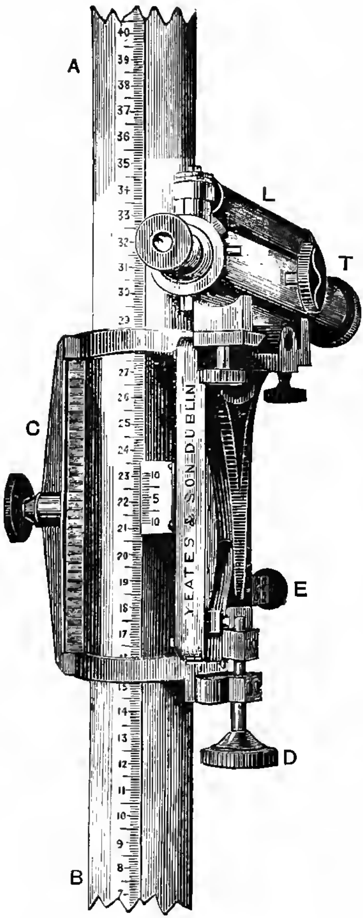


Fig. 9.

In using the cathetometer it is essential to remember that the axis of the telescope must always remain at the same angle to the axis of the vertical rod carrying the scale. It is most convenient to make this angle a right angle, that is to say, to make the rod vertical and the axis of the telescope

to the fact that the image does not coincide with the plane of the cross hairs, or of the scale against which it is to be read.

horizontal.* This is easy enough to adjust for one position; the difficulty is to preserve this exact relation when the telescope is moved vertically up or down, for it is seldom that the vertical support is absolutely true throughout its length; it is therefore necessary accurately to adjust the level of the telescope before each observation by means of the screw E and graduated level on C. Any error in the levelling of the telescope will obviously be more serious the *farther* the instrument is distant from the object measured; hence this distance should be made as small as possible.†

In making the experiment with the cathetometer the telescope is focused on one mark, and the fine adjustment made by the screw D; the scale and vernier are then read off. The carriage with the telescope is now slid up or down the vertical, and the telescope similarly focused on the other mark, and the reading again taken. The difference between these two readings is the vertical distance between the two marks in scale divisions of the cathetometer.

Example.—Find by means of the cathetometer the distance between the 20th, 30th, and 40th divisions on the centimetre scale.

The cathetometer has a millimetre scale on the stem, and the "least count" of the vernier is ·05 mm. (Fig. 9).

Enter results thus :—

* It is *essential* to do this when the two marks observed are not at the same distance from the instrument.

† See description of cathetometer microscope and reading telescope in Appendix, § 3.

Reading on the Scale in cms.	Reading on the Cathetometer in cms.	Difference.
20·00	40·43	...
30·00	30·41	·02
40·00	20·42	·01

Exercise.

Compare the millimetre scale of the cathetometer with a scale of inches, and find the factor for converting inches to cms., and *vice versâ.*

Experiment 7.—**To construct and etch a millimetre scale on glass.**

Instruments required.—Strip of plate-glass, a standard scale, and beam compass; also hydrofluoric acid.

Coat a warm, dry glass strip with bees'-wax, in which a little turpentine has been mixed, fasten it to the bench with soft wax, the coated side uppermost. End to end, about 10 cms. off, similarly fasten the standard scale, which may be a steel rule 20 to 30 cms. long, divided into centimetres and millimetres. Place one point of the beam compass on any division of the standard scale, and with the other point draw a line on the coated glass surface, cutting through the wax; then lift the points, and repeat the operation at the next division of the scale, and so on. To ensure regularity in the length of the divisions, fix a thin strip of brass or stout tinfoil over the coated glass strip, which will serve as a stop or guide to the marking point; for the larger marks at the fifth and tenth divisions

corresponding niches can be made in the guide strip, or these divisions may be omitted in the first instance, and put in after the metal strip has been set farther back. Neatly write the numbers on the scale with a fine dry steel pen. Carefully inspect and, if necessary, test by compasses the scale when made; if any mistakes have been made re-melt the wax on the bad spot by means of a hot wire and mark it again; if found correct then etch in the divisions as follows: Lay a strip of blotting paper on the whole length of the glass, and pour on it a little dilute hydrofluoric acid, which can be obtained in solution, in small gutta-percha bottles; or tie a tuft of cotton wool to a stick, moisten it with hydrofluoric acid, and gently dab on the waxed surface. Breathe on the scale before applying the acid, or the latter may not bite. The fumes of hydrofluoric acid gas, liberated by pouring strong sulphuric acid on some powdered fluorspar contained in a lead trough, which is gently warmed, produce opaque and easily-read markings.

After exposure to the acid fumes or to the liquid acid for a few minutes, wash carefully and clean off the wax. To render the markings more visible, it may be necessary to rub over the whole with cotton wool moistened with Brunswick black, or coloured paint, and then wipe off with glazed notepaper or a smooth stick. The colour remains in the markings, and when dry is fairly durable.

Another plan, adopted by scale-makers, is to use a ruler of the shape of a right-angled triangle, and lay it over the standard scale and the scale to be made, which

must, in this case, be fixed side by side. One edge of the ruler has a ledge which is pressed against the side of the standard scale, so that the markings can be made parallel to each other. A small point is fixed to and projects below the right-angled ruler; this is allowed to drop into the divisions of the standard scale and thus acts as a guide. By means of a needle fixed to a small handle, or a fine pen held perpendicularly and steadily against the edge of the ruler, scratch the divisions on the wax surface: when finished etch in as before.

Experiment 8.—**Construction of a scale by means of the dividing engine.**

Instruments required.—The dividing engine and a strip of glass.

The dividing engine is a valuable but costly piece of physical apparatus, and acquaintance with its construction and use is far better obtained by examination of the instrument than by any description. Note the following points (i.) The pitch of the screw, one revolution of which drives the carriage carrying the marking point through a known distance. (ii.) The number of divisions of the large divided circle attached to the screw. (iii.) The means of adjusting the range of motion of the handle to any required graduation. (iv.) The arrangement of the pawl and ratchet on the carriage, whereby the movement of the marking point rotates a notched wheel that enables every fifth mark to be longer, and every tenth longer still; or, by another notched wheel every second and fourth division to be longer, according to the scale we

require to construct. To divide a space between two marks into a given number of equal divisions, such as 100 or 180, as in the construction of a thermometer scale, bring the carriage and marking point to the first mark, and read the position of the screw, then turn the handle till the second mark is reached and again read the number of turns and parts of a turn of the screw included between the two marks. Divide this number by 100 or 180, as the case may be, and set the instrument to turn through this range. Now lift the carriage and bring it back to the first mark, adjust accurately by small motions of the handle of the screw until the marking point exactly coincides with the first mark, and proceed to divide the waxed surface, etching in afterwards as explained in the previous experiment.

SECTION II.—*Angular Measurement.*

Experiment 9.—**The measurement of angles.**

Instruments required.—A pair of compasses, a protractor, a millimetre scale, and a scale of chords.

(1) By the Protractor.

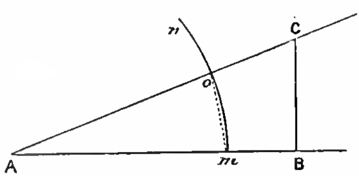


Fig. 10.

In Fig. 10, if we want to measure the angle A, we put the centre point of the protractor on the point A and its base along AB, then read off the division on the protractor which coincides with the

direction of the line AC, which will be the angle required in degrees.

(2) By the Scale of Chords.

Take off on a pair of compasses the distance from 0 to 60 on a scale of chords, and with A as centre describe the arc of a circle *mn*, then take off the distance *mn* in the compasses and apply it to the scale of chords, one point of the compasses being at zero, when the reading of the other point will be the required angle in degrees.

(3) By Trigonometrical Ratios.

At any point B in AB draw a perpendicular, meeting AC at C, and measure off BC and AB with the compasses and a millimetre scale, then $\tan BAC = \frac{BC}{AB}$ (Fig. 10), and by consulting a table of natural tangents we get the required angle. This method of measuring angles is of frequent occurrence in magnetic and electric measurements; thus in a mirror galvanometer, since the angle which the reflected beam is turned through is twice that of the mirror itself, then d the deflection on the scale, divided by D the distance from the scale to the galvanometer mirror, is the tangent of twice the angle of the mirror.

That is

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{d}{D},$$

$$\therefore \tan\theta \fallingdotseq \frac{d}{2D}.^*$$

* The sign $\fallingdotseq$ denotes approximate equality.

Exercises.

1. Draw by means of the protractor angles of 24° and 63° and measure them, (*a*) by the scale of chords; (*b*) by trigonometrical ratios.

2. Determine the angle through which the mirror of a reflecting galvanometer moves when it has turned the spot of light through 100 scale divisions, the scale being 100 cms. from the mirror.

Experiment 10.—**To determine a small thickness by means of the optical lever.**

Instruments required.—An optical lever, a lamp, and scale.

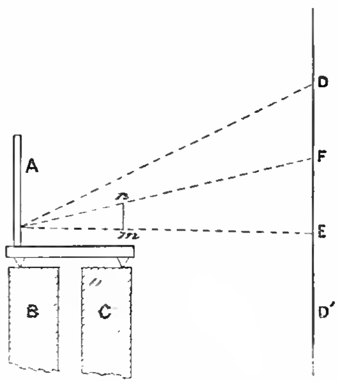


Fig. 11.

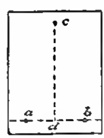


Fig. 12.

A modified form of Cornu's optical lever (Fig. 11) consists of a small rectangular mirror A fixed at right angles to a base piece. On the under side of this base are three small conical toes, shown in section in Fig. 11 and in plan at *a*, *b*, *c* in Fig. 12.

B and C are two rigid supports, with plane level surfaces, on which the optical lever rests during an experiment. A shallow V-groove is cut along B, in which the two toes a and b (Fig. 12) rest; the other toe c rests on the plane surface C. DD′ is a lamp and scale, or telescope and scale arrangement for measuring the deviation of the mirror during an experiment.

Three measurements are required in the experiment: (i.) the line cd (Fig. 12); (ii.) the distance in corresponding units of AE; and (iii.) in similar units the scale reading DE. An easy way of obtaining the first is to gently press the toes of the optical lever on to a piece of paper or card, and measure by means of the impressed pricks.

To measure a thin parallel plate it is put on C underneath the conical toe, and the beam of light which was reflected (back along its own path) from the mirror to E on the scale is now reflected to D, and the deflection DE is read off.

To measure a small increment of length, such as the expansion of a given bar C, a micrometer screw (not shown in the figure) is used to bring the mirror to the zero of the scale, and the experiment then made.

Let x = thickness or increment of length required,
D = the distance from the mirror to scale,
d = *half* the deflection of the reflected beam,
l = the distance cd in Fig. 12 between one foot C and the line joining ab,

then, since D = AE, d = DF, $x = mn$, by similar triangles we get—

$$\frac{x}{l} = \frac{d}{D}, \quad \therefore \; x = \frac{ld}{D}.$$

Example.—Find the thickness of a piece of microscopic glass by means of the optical lever.

$$D = 320 \text{ cms.}, \quad d = 10 \text{ cms.}, \quad l = 1 \text{ cm.}$$

$$\therefore \; x = \frac{10 \times 1}{320} = {\cdot}0312 \text{ cms.}$$

By measurement with the screw-gauge the thickness of glass was found to be ·031 cms.*

Exercises.

1. Find the thickness of a plate of mica.

2. By means of the micrometer screw at the lower end of the support C find what number of scale divisions correspond to a tenth of a millimetre when the scale is 5 metres from the mirror.

Experiment 11.—**Measurement of vertical heights by the sextant.**

Instruments required.— Sextant and a measuring tape. The sextant (Fig. 13) consists of a graduated circular arc AB of about 60°, joined to the centre D by the two rigid arms BD and AD. DC is a third arm, movable round the centre D, and having at one end C a vernier with tangent screw attached, and at the other a mirror fixed at right angles to the plane of the scale. E is the horizon glass, having

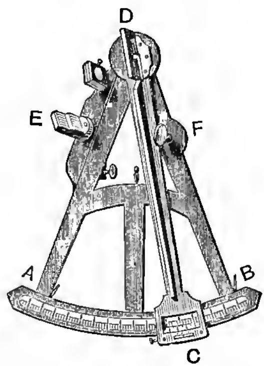


Fig. 13.

* The "horizontal pendulum" affords another delicate method of measuring minute displacements, Appendix, § 4.

one half silvered and the other half unsilvered, so that the eye on looking through a small hole in the disc F can see an object directly through the unsilvered part and simultaneously the reflection of another object from the silvered part.

Since in a rotating mirror the angle through which the reflected beam of light is turned is twice that of the mirror itself, the graduations on the scale are purposely marked double of what they really are, so as to enable the scale to be read off directly in an observation.

In using the sextant to measure a vertical height, we first measure off a vertical distance equal to the height of the eye of the observer, and marking this point make it our horizon. Then look through the hole in the disc F directly at the horizon mark (some instruments are furnished with a telescope and cross hairs instead of a disc at F) and at the same time move the arm DC till the image of the summit whose height we are measuring, after being reflected from D and E, coincides with the horizon mark as seen directly. Then the reading on the scale and vernier gives the angle subtended by the object at the eye of the observer.

Knowing this angle and measuring by a tape the distance from the object to the observer, we can calculate the height required, and by adding to this the height of the horizon mark, we get the total height.

The zero of the sextant should always be tested before making an observation, and this is done by clamping the vernier at zero, and looking at a *distant* object, such as a horizontal window bar, when, if the zero be correct, the

part of the bar as seen directly will be continuous with the part seen by reflection.

If they do not coincide a small motion of the horizon glass round its axis makes this adjustment complete; that is to say, the horizon glass and the mirror D are now parallel, and both are perpendicular to the plane of the scale, which latter condition is necessary for the reflected ray to have an angular motion twice that of the mirror.

Example 1. — To find the height of a building, whose *base is accessible,* by means of the sextant.

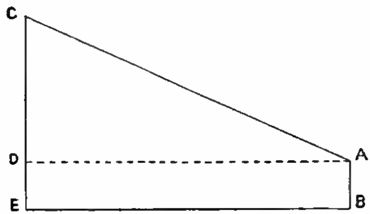


Fig. 14.

In Fig. 14 the height of the observer's eye $AB = ED = 5$ feet, which is marked off.

The base $BE = AD = 110$ feet.

The angle $CAD = 19^\circ\ 15'$;

$$\therefore\ CD = AD \times \tan CAD$$
$$= 110 \times \cdot 3492 = 38 \cdot 41 \text{ ft.};$$
$$\therefore\ CE = 38 \cdot 41 + 5 = 43 \cdot 41 \text{ ft.}$$

The height by direct measurement was found to be 43·5 feet, hence the error in the calculated height is about 1 inch, or 0·2 per cent.

Example 2.—To find the height of a building with *inaccessible base* (in this experiment a sunk area).

In Fig. 15 measure the distance BG, and observe the angles DCE, DAC, DCF. Thus the base—

$$BG = 30 \text{ ft.}, \quad DCE = 28° \ 45',$$
$$DAC = 16° \ 5', \quad DCF = 45° \ 55'.$$

Then the angle $ADC = 28° \ 45' - 16° \ 5' = 12° \ 40'$;

$$\therefore \ DC = \frac{30 \times \sin 16° \ 5'}{\sin 12° \ 40'} = \frac{30 \times \cdot 277}{\cdot 2192},$$

and $$DE = DC \sin 28° \ 45' = DC \times \cdot 481;$$

$$\therefore \ DE = \frac{30 \times \cdot 277 \times \cdot 481}{\cdot 2192} = 18 \cdot 24 \text{ feet.}$$

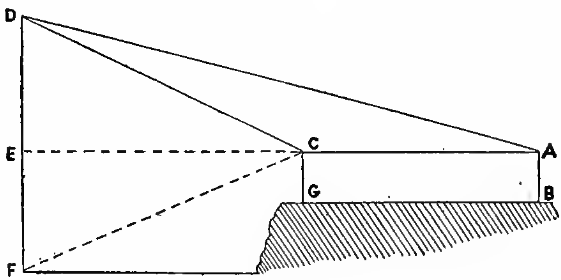


Fig. 15.

Then the width of the area (required to find EF)

$$CE = \frac{18 \cdot 24}{\tan 28° \ 45'} = \frac{18 \cdot 24}{\cdot 5486} = 33 \cdot 25 \text{ feet},$$

and $$EF = CE \times \tan ECF,$$

but $$ECF = DCF - DCE$$
$$= 45° \ 55' - 28° \ 45' = 17° \ 10';$$

$$\therefore \ EF = 33 \cdot 25 \times \cdot 3067 = 10 \cdot 2 \text{ ft.}$$

Therefore the total height DF is

$$18 \cdot 24 + 10 \cdot 2 = 28 \cdot 44 \text{ feet.}$$

The height by direct measurement was found to be 28·3 feet, hence the error in the calculated height is 1·68 inch, or 0·5 per cent.

Exercises.

1. Find the height of the gutter of the laboratory roof from the ground.

2. Find the height of a house with a street intervening when the traffic on the street prevents direct measurement of its width; calculate its width.

Experiment 12.—**Approximate measurement of heights by simple methods.**

(1) Cut an inch square hole through a piece of wood an inch thick. Fig. 16 shows a section of the hole. AB is the height to be measured.

The eye being placed at C, walk backwards or forwards till the ray of light CA from the top of the object is seen to graze the upper edge D, the bottom B of the object being also seen along the lower side of the hole CE.

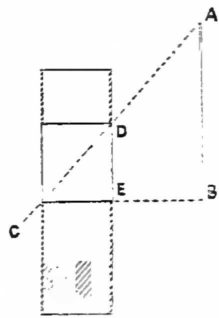


Fig. 16.

Then the distance from the observer to the object is the height required, because the angle DCE = 45°.

(2) The same result may be obtained by using a card or piece of wood cut in the shape of a right-angled triangle with equal sides. The eye is placed at one of the acute angles, the bottom of the object being viewed along the base and the top along the hypothenuse: then proceed as above.

(3) In sunlight put up a rod of known length and measure its shadow, then measure the length of the shadow of the object whose height is required. If l = length of the rod, s = length of its shadow, L = height required, S = length of shadow of height required, then, by similar triangles,

$$L = \frac{lS}{s}.$$

(4) In Fig. 17 AB is the height to be measured. On

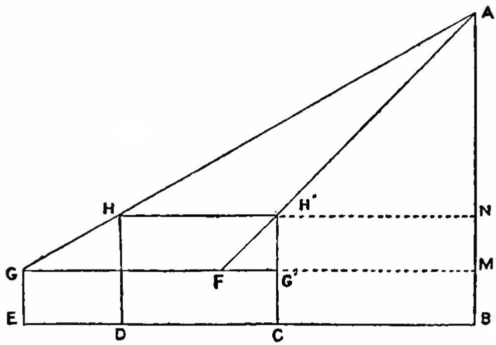


Fig. 17.

the level ground BE measure off a convenient distance DC, and place rods CH′ and DH at each extremity of the length CD.

CG′ = EG, the height of the eye of the observer. Let F be the position of the eye when the top of the object A is just seen over the top of the rod CH′, and measure FG′.

Then find another position of the eye G where the same point A is just seen over the top of the second rod DH, and measure DE.

Now let us call

$$CD = HH' = d, \quad FG' = a, \quad DE = b,$$

$$CG' = c, \quad DH = h, \quad \text{and } AB = H,$$

then
$$H = \frac{d(h-c)}{b-a} + h.$$

This formula is proved by aid of the similar triangles AGF and AHH′, thus

$$\frac{d}{b+(d-a)} = \frac{H-h}{H-c},$$

$$H(b-a) = d(h-c) + h(b-a),$$

$$H = \frac{d(h-c)}{b-a} + h.$$

SECTION III.—*Area Measurement.*

Experiment 13.—**Measurement of the area of a plane surface.**

Instruments required.—Millimetre paper, millimetre scale, compasses, scissors, and ruler.

First method.—From geometrical dimensions (see Table III.)

Second method.—Transfer the figure whose area is required to curve paper, and count all the included areas, estimating by eye the decimal parts of the small areas just round the boundary.

Third method.—Transfer the figure to stout paper or tinfoil of uniform thickness, then cut the figure out and weigh it; also cut out of the same paper or foil a *known* area and weigh it; then the ratio of these two weights will be the ratio of the two areas.

Example.—Find by the three methods the area of a triangle whose sides are 6, 9, 11 cms. respectively (see Table III., 4).

$$S = \tfrac{1}{2}(6 + 9 + 11) = 13;$$

$$\therefore A = (13 \times 7 \times 4 \times 2)^{\frac{1}{2}} = \sqrt{728} = 26{\cdot}98 \text{ sq. cms.}$$

Method.	Area. Sq. cms.	Per Cent. Error. (See next paragraph.)
First . .	26·98	0
Second . .	26·72	0·96
Third . .	26·63	1·22

Assuming the result obtained by the first method to be correct, the percentage error by the second method is obtained thus

$$26{\cdot}98 - 26{\cdot}72 = 0{\cdot}26;$$

$$\therefore 26{\cdot}72 : 100 :: 0{\cdot}26 : x,$$

$$x = 0{\cdot}96.$$

In like manner the percentage error by the third method is found to be 1·22.

Exercises.

1. Find by the three methods named the area of a circle 10 centimetres in diameter.

2. Find by the three methods the area of the remainder obtained by cutting the inscribed circle out of a square of 8 cms. in the side.

Experiment 14.—**Measurement of areas by the planimeter.**

Instrument required.—Amsler's planimeter.

In the planimeter, of which the form devised by

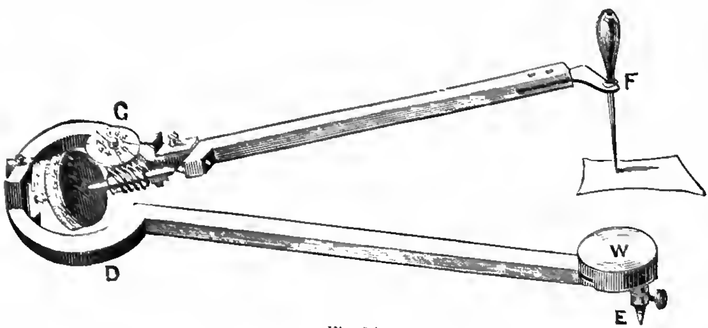


Fig. 18.

Amsler is shown in Fig. 18, a point is made to traverse the boundary of the surface the area of which has to be measured, and the required area is given directly by reading off the graduated rim of a wheel. As usually constructed, the instrument reads to square inches and hundredths of a square inch, or square centimetres and hundredths of a square centimetre.

To find the area of any surface, place the instrument on the drawing or the irregular surface to be measured, with the tracing point F at a mark on the curve. Press the needle point E slightly into the paper anywhere *outside* the curve to be measured. Read the roller above the letter D and the disc G; say the reading is 2·368, *i.e.* 2 from the disc G, 36 from the roller, and 8 from the vernier; note this down. Now carefully move the tracing point all round the area in the direction of the hands of a watch, and when the starting-point is reached, take the reading again; subtract the first reading from the second, and the difference multiplied by 10 will be the exact area in square inches and hundredths of a square inch.

A weight W keeps the fixed point E steady; this point is fixed in such a position that the point F in a preliminary trial can travel completely round the area to be measured.*

Exercises.

1. Repeat the measurement given in first exercise of Experiment 13 by means of the planimeter.

2. Find the area of Phœnix Park from the Ordnance Map of Dublin.

Experiment 15.—**The graphical representation of experimental results.**

Instruments required.—Millimetre paper and a flexible strip of wood for drawing the curves.

* The theory of the planimeter is given in Williamson's *Integral Calculus*; or in Professor Hele Shaw's paper on "Mechanical Integrators," *Proc. Inst. Civil Engineers*, 1885.

The object of a graphic representation is to show to the eye the relation between any two quantities x and y connected in such a way that a change in the one alters the other.

To plot a curve on the millimetre paper take the intersection of a horizontal and vertical line as the origin, and lay off the determined values of x along the horizontal line, which is called the axis of abscissæ, and the corresponding values of y along the vertical, called the axis of ordinates; then the smooth curve drawn through these various points will represent graphically the relation between the two quantities x and y.

In plotting the results of an experiment it will be found that a smooth curve will not always pass through the marked points; this is usually due to accidental errors in the experiment, but in every case the judgment must be used in drawing the smooth curve so as to represent as nearly as possible the actual results of the experiment.

It is not necessary that the same scale should be used for both the horizontal and vertical distances; this should depend on the size of the paper. By making the curve as large as possible small errors in observation are more easily indicated.

This graphic method is of great importance in all physical work, and will be much used throughout this book; we therefore append some examples for the student to practise.

Example 1.— Plot the curve whose equation is $xy = 128$.

This is an equilateral hyperbola, and is approximately

the curve obtained when volumes of air are noted under varying pressures and the results plotted.

Give various values to x, and find by calculation the corresponding values of y; tabulate the results thus:—

x	y	x	y
1	128·00	8	16·00
2	64·00	10	12·80
4	32·00	12	10·66
6	21·33	14	9·14

Now lay off on the horizontal axis the values of x, and on

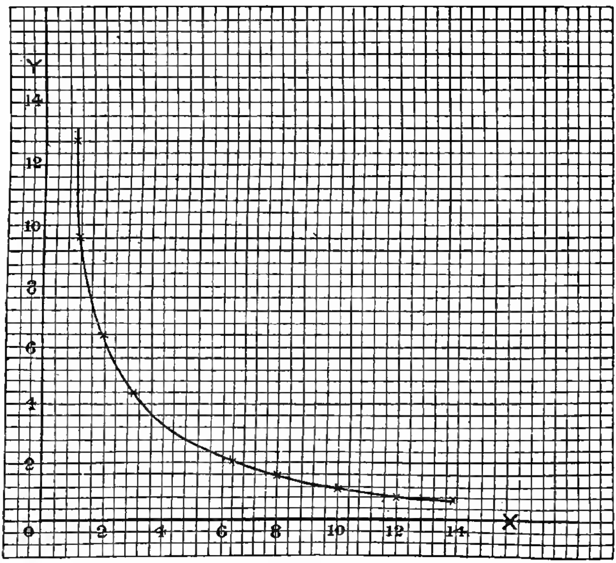


Fig. 19.

the vertical the corresponding values of y, divided by 10 for convenience, as shown in the curve, Fig. 19.

Example 2.—Plot the curve whose equation is $y = x^2 + 4x - 2$.

Give various values to x and calculate y as before, thus:—

x	y	x	y
0	$-\ 2$	-3	$-\ 5$
1	$+\ 3$	-4	$-\ 2$
2	$+10$	-5	$+\ 3$
3	$+19$	-6	$+10$
-1	$-\ 5$	-7	$+19$
-2	$-\ 6$	-8	$+30$

On plotting these values the curve will be seen to be a *vertical* parabola with vertex downwards, the co-ordinates of the vertex being $(-2, -6)$.

Exercises.

Plot the curves whose equations are—

(1) $y = \frac{1}{2}x + 2$.

(2) $y^2 = 8x$.

(3) $y = 2 \pm (25 - x - x^2)^{\frac{1}{2}}$.

SECTION IV.—*Volume Measurement.*

Experiment 16.—**Determination of volume.**

Instruments required.—Calipers, a vessel with overflow, a beaker, clean mercury, and a balance.

First method.—If a solid body is of regular geometrical shape, take its dimensions with the calipers and calculate its volume (see Table III.)

Second method.—If the body has an irregular shape, take a vessel with a lip or overflow and fill it with water just up to the overflow; then immerse the body whose volume is required in the vessel and it will displace its own volume of water, which can be caught in a beaker. Now weigh the displaced water in grammes, and the number of grammes weight will be the number of cubic centimetres in the body, which can be corrected for temperature if necessary (see Table VIII.)

Example.—Find the volume of an ivory ball 4 cms. in diameter by both methods.

$$(1) \quad V = \frac{4}{3}\pi r^3 = \frac{4 \times 3{\cdot}1416 \times 2^3}{3} = 33{\cdot}5104 \text{ c.c.}$$

(2) The weight of the water at 15° C. displaced by the ball = 33·483 grammes.

Volume = 33·483 c.c.

Method.	Volume.	Per Cent Error. (Page 38.)
Calculation . .	33·5104	0
Displacement . .	33·483	·08

Note.—The weight of the body in air divided by the volume of the body thus found is a simple way of determining the specific gravity of an insoluble body (p. 62 *et seq.*)

Third method.—To find the internal capacity of a flask up to a certain mark on the neck. If the flask is

small use pure mercury, and if large use distilled water. Then if

W = weight in grams. of the flask when clean and dry,
W' = weight of the flask when filled with liquid up to the mark,
ρ = density of the liquid at t° C.

Then $$W' - W = \text{the weight of the liquid};$$

$$\therefore \text{Volume} = \frac{W' - W}{\rho}.^*$$

Exercises.

1. Find the volume of a ball and of a cube by calculation and displacement.

2. Find the internal capacity of a specific gravity flask.

Experiment 17.—**Volume, radius, and calibration of a narrow tube.**

Instruments required.—Pure mercury, a watch-glass, a millimetre scale, and a balance.

Introduce a short thread of mercury into the tube, and measure its length when it is at various positions in the tube, which will indicate the uniformity or otherwise of the bore of the tube.

Let the mercury run out into a watch-glass and weigh it. Then if

* The volume of a body can also be found by the hydrostatic balance or by the Stereometer (see Experiments 23 and 33).

$M =$ weight in grammes of the mercury thread,
$l =$ mean length of the thread,
$r =$ mean radius of the tube,
$\rho =$ density of mercury at $t°$ C. (see Table IX.),

Volume $= \pi r^2 l$,

Mass $M = \pi r^2 l \rho$.

$$\therefore \text{ Sectional area} = \pi r^2 = \frac{M}{l\rho},$$

and
$$r = \sqrt{\frac{M}{\pi l \rho}}.$$

Example.—The mean length of the mercury thread in a tube $= 5{\cdot}41$ cms. and the weight $= {\cdot}743$ gramme, the temperature 15° C.

$$\text{Sectional area} = \pi r^2 = \frac{{\cdot}743}{5{\cdot}41 \times 13{\cdot}56} = {\cdot}01 \text{ sq. cm.}$$

Exercises.

1. Ascertain whether the bore of the capillary tube given you be uniform.
2. Find the mean radius of a capillary tube.

Note.—The *reading microscope* also affords a rapid and accurate way of directly measuring the bore at the end of a capillary tube and the diameter of a fine wire, when the cross-section of each is circular. A description of this instrument is given in Appendix, § 3, which the student should read. In using this method care must be taken to ascertain the uniformity of the section of the tube by a mercury thread as above.

Experiment 18.—**To make and graduate a Burette.**

Instruments required.—About 3 feet of stout lead or soda glass tubing about 1 or 1·5 cm. internal diameter, and a blow-pipe flame.

A burette is a graduated tube for the delivery of known quantities of a liquid, and its construction affords useful practice in simple blow-pipe work.* To make a burette, heat the tube uniformly in the blow-pipe flame by turning it round all the time; when the glass is soft remove the tube from the flame and draw it steadily apart. After the heated part has cooled slowly, with a triangular file scratch the glass at the contracted portion, and it will easily snap across.† Fuse the edges of the tube at each end, and when the tube is cold slip on the contracted end a short piece of rubber tubing, nipped by a pinch tap or plugged by a bit of glass rod. Coat the tube with melted bees'-wax, and having fixed it in a clip, pour into it 10 grammes of water, mark the level of the water in the tube, now add another 10 grammes, and make another mark, and so on till the tube is nearly full. Now fasten the tube to the dividing engine,‡ and set the instrument so that 10 divisions can be made in the first marked space; if the marks are equidistant,

* The student who is unfamiliar with glass-blowing should begin by bending tubes in an ordinary flat luminous gas flame, then proceed to practise drawing out tubes, then sealing platinum wire into tubes, and afterwards blowing bulbs and making glass T-pieces. Full and excellent instructions are given in Mr. Shenstone's *Methods of Glass Blowing* (published by Rivingtons), which the student will do well to procure.

† A knife, made glass-hard by plunging in cold water when bright red-hot, is better than a file for cutting glass.

‡ If the student has not access to a dividing engine, the method described in Experiment 7 can be followed for marking in the divisions.

continue the division along the whole tube. Probably it will be found that the length between the marks is not quite the same; in this case the dividing engine must be set for each pair of marks.* Now write the corresponding number on the wax, beginning at 0 at the uppermost division near the mouth of the tube, and putting a figure at every 5th or 10th division. Etch in the divisions and figures as already explained (Experiment 7), and remove the wax. If all has gone well, proceed to *calibrate the tube* by filling it with distilled water, drawing off, say, 5 c.c. at a time and weighing the quantity. After repetition enter the mean weights found in a table, thus:—

0 – 5	c.c. = 5·10	grammes.	
5 – 10	c.c. = 5·00	,,	
10 – 15	c.c. = 4·92	,,	etc.

Use this table to estimate the true volume of the liquid employed, when say 20 c.c. are drawn off between the divisions 20 and 40. For accurate purposes a correction must be made for the temperature of the water (Table VIII.), since at 4° C. 1 gramme of water has a volume of 1 c.c. In reading the level of the water in the burette, hold a piece of white paper behind and read from the lower edge of the meniscus—a black line will there be seen.

* The sub-division of a given length can also be done by means of a "diagonal-scale." A very simple form of Line-divider is described by Miss Marks in *Proc. Physical Soc.*, February 1885.

CHAPTER III

MEASUREMENT OF TIME

THE unit of time has been already defined as the second or the 1/86400 part of the mean solar day.

The length of a simple pendulum which beats seconds at every single oscillation is 99·43 cms. or 39·14 inches in Dublin. In the accurate estimation of time various methods may be followed, according to the nature of the experiment and the degree of accuracy required.

Experiment 19.—**Find the time of oscillation (half-vibration period) of a simple pendulum by the 'method of passages.'**

Instrument required.—A heavy bullet suspended by a fine silk thread.

The method of passages consists in finding, at first approximately, the time of oscillation by noting the exact moment when the middle of each swing occurs during, say, 5 consecutive oscillations; then determining the exact number of oscillations in a longer interval by dividing this interval by the approximate oscillation period previously obtained, and selecting the nearest integer. This gives a closer approximation. A still longer interval

may now be selected and divided by the last approximation, the nearest integer being selected as before. The whole time being divided by the integer so found gives the true time of a single oscillation.

Example.—A heavy bullet was suspended by a fine silk thread and allowed to vibrate through a small arc.

The transits of the thread across the wire of a telescope were noted and the time taken by listening to and counting aloud the beats of the seconds pendulum of a standard clock.

The following results were obtained :—

Transit.	Time.	Transit.	Time.	Difference.	Time of a Single Transit.
	Min. Sec.		Min. Sec.	Secs.	
0	15 5·0	25th	15 28·0	23	0·92
5th	15 9·5	30 „	15 32·5	23	0·92
10 „	15 14·0	35 „	15 37·0	23	0·92
15 „	15 18·5	40 „	15 42·0	23·5	0·94
20 „	15 23·5	45 „	15 46·5	23	0·92

Mean 0·924

Subtracting the time at 0 from that at the 25th transit we get the time required for 25 transits, and so on for each pair, as given in the column of differences.

By dividing each difference by 25 we get the time of a single transit, and the mean gives 0·924 of a second, which is the first approximation.

From this the time taken for 100 oscillations should be $100 \times {\cdot}924 = 92{\cdot}4$ seconds. Now observe the seconds hand of the clock as it comes to 60, and count aloud from this point, watching the transits of the pendulum; the

100th transit occurred at 93·0 secs., and dividing by 100 we get 0·93 of a second as the second approximation. Again estimate from this the time of 500 oscillations, this comes to 7 minutes 45 seconds. Leave the pendulum swinging and return to the observation at 7 minutes 40 seconds; counting the beats of the clock from this instant, the transit occurred at 7 minutes 45 seconds very approximately.

Dividing this by 500 we get 0·930 of a second as the true time of an oscillation.

Exercise.

Make an experiment similar to the above.

Experiment 20.—**Determination of the period of oscillation by the 'method of coincidences.'**

Instrument required.—A clock with seconds pendulum and a simple pendulum hung in front of the clock.

Suspend the simple pendulum used in the last experiment in front of the seconds pendulum of the standard clock, making the length of the suspending thread slightly greater than before, so that its period shall coincide as nearly as possible with that of the clock pendulum. It is best to start the pendulum by pulling it aside with a thread, which is then burnt; this avoids giving any rotatory motion. If the period of the simple pendulum be now slightly greater than a second, a moment will occur when the beat of both pendulums will precisely coincide; the clock will then gain on the experimental pendulum, and

after a certain interval it will gain a complete oscillation, when coincidences will again occur. The interval of time between the two moments of coincidence is to be accurately noted. The same thing will occur if the experimental pendulum be slightly quicker than the clock. In this case, if n be the interval between two successive coincidences, the experimental pendulum will in n seconds have made $n+1$ oscillations, and the time of each oscillation will therefore be $\frac{n}{n+1}$ seconds. In the former case the experimental pendulum will have made $n-1$ swings in n seconds, and the time of its oscillation will be $\frac{n}{n-1}$ seconds.

This method, it is obvious, can only be used when the time of vibration is very nearly alike in the two cases.

Example.—Find the time of oscillation of a simple pendulum by the foregoing method.

The experimental pendulum was slightly quicker than the clock pendulum.

The mean of five coincidences gave $n = 21$ seconds. Hence the time of oscillation was $0{\cdot}9\dot{5}\dot{4}$ of a second.

Exercise.

Repeat the above experiment, making the experimental pendulum first slightly longer then slightly shorter than the clock pendulum.

Experiment 21.—**Determination of small intervals of time.**

Instruments required.—A stop-watch, an ordinary watch, a clepsydra, a chronoscope, and a tuning-fork chronograph.

(1) The seconds hand of a stop-watch traverses the whole dial in one minute and is made to indicate fifths of a second. There is a slight loss of time in starting the watch, but a similar one in stopping it, so that the error is extremely small. For most experiments in timing oscillations as on the value of gravity, a stop-watch is the most convenient instrument to use.

(2) An ordinary watch generally gives five ticks to a second, with a little practice fractions of a second may be accurately estimated by this means. Proceed as follows: make a series of twenty-five strokes on paper, put an ordinary watch to your ear and listen till you get accustomed to the sound of every second tick; now holding a pencil in your hand ask a second observer who is holding a stop-watch to start it directly you point to the first stroke, and stop it when you reach the last, you meanwhile pointing to the successive strokes at every second tick, or two-fifths of a second. In this way you will, after one or two trials, find 10 seconds accurately registered in the interval, indicated by the twenty-five strokes. Now try in the same way 30 seconds. By this preliminary practice with a stop-watch you will thus obtain confidence in your ability to count the ticks, which at first seems a hopeless undertaking. For a short interval of time, such as that taken by a falling body, an ordinary watch can be used to replace a stop-watch.

(3) The flow of water through an orifice, under a constant head, also gives a means of estimating time. For this purpose the water is caught in a vessel at the commencement of the period, and the vessel promptly withdrawn at the end, the quantity of water in the vessel is then weighed. The time is now taken which corresponds to the flow of any given quantity of water, and the interval of the time occupied by the experiment is thus estimated, the time being proportional to the quantity of water, all other conditions remaining the same.

(4) A falling weight which actuates clockwork can also be used to measure small intervals of time up to 1/300th of a second. This form of chronoscope is most easily started and stopped by an electro-magnetic arrangement. A preliminary experiment is made in timing the instrument, the rate of which can be varied by adding shots to the bucket which forms the falling weight. A complete revolution of one hand on the chronoscope being made to correspond to one second, each revolution of the other and smaller hand corresponds to 1/30th of a second, and as this dial is divided into ten parts, the 1/300th of a second can easily be estimated.

(5) The most accurate method of registering small intervals of time is by means of a tuning-fork chronograph. This method will be described in Part II in the section on Sound. The fork, which is kept vibrating by an electro-magnet, has a style (a) attached to one prong, which writes upon a rotating blackened cylinder a sinuous curve resulting from the vibration of the fork. Its exact period of vibration is determined by a current from a

seconds pendulum electro-magnetically moving another style, which makes a record on the blackened cylinder. When an experiment is to be made, such, for example, as the time taken by a freely falling body, the beginning and the end of the fall are indicated on the rotating cylinder—simultaneously with the record made by the fork—by the falling body making and breaking an electric circuit, which actuates another style (b); or an induction coil may be used, the primary circuit being made and broken, and the spark from the secondary marking the blackened cylinder. This has some advantages over the use of a style.

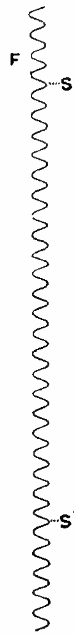


Fig. 20.

This last experiment is well suited for lecture demonstration, as the moving styles and their record can be easily projected on the screen, the tuning-fork being mounted vertically—a vertical strip of blackened glass sliding in a grooved upright being used instead of the rotating cylinder. The following is the result of an actual lecture experiment:—

Example.—A small brass weight was fixed at the height of 8 feet 7 inches above a hinged shelf forming a contact breaker. The electric circuit comprised the movable jaw which clipped the weight, the electro-magnet actuating the style b and the contact breaker below. The tuning-fork gave 30 vibrations per second recorded on the strip of blackened glass by the style a. The glass was raised by hand, the styles a and b lightly pressing upon it. On raising the glass an

assistant released the weight by pulling a string which opened the jaws of the clip and simultaneously made contact; the style b thereupon recorded this instant. On the weight striking the hinged lid contact was again broken and the style b moved. The sinuosities made on the blackened glass by the style a are shown at F, and the marks by the style b are shown at SS′, on Fig. 20. The number of sinuosities between S and S′ was $22 = \frac{22}{30}$ of a second. From this the height of fall h can be calculated (see Experiment 49).

$$\begin{aligned} h &= \tfrac{1}{2}\, gt^2 \\ &= 16{\cdot}1 \times \left(\tfrac{22}{30}\right)^2 \quad \text{(in feet)} \\ &= 8 \text{ ft. } 6{\cdot}7 \text{ in.,} \end{aligned}$$

the actual height being 8 ft. 7 in.

CHAPTER IV

MEASUREMENT OF MASS—SPECIFIC GRAVITY

Experiment 22.—**The comparison of masses by means of the chemical balance.**

Instruments required.—A balance and box of gramme weights.

The chemical balance (Fig. 21) consists essentially of a metal beam A supported at the middle on a knife-edge so as to be free to move in a vertical plane round an axis perpendicular to its length. At the end of this beam there are knife-edges BB′ for supporting the scale pans SS′, which can turn freely round axes parallel to the axis of rotation of the beam. These axes are usually formed of agate knife-edges resting on plates of agate. To prevent injury when the balance is out of use, or when the weights are being changed, an "arrestment," actuated by a handle D outside the balance case, lifts the knife-edges off the agate planes.

A long pointer is fixed to the beam of the balance with its length perpendicular to the line joining the extreme knife-edges, and serves to define the position of the beam as indicated in the short scale fixed to the pillar of the balance.

When the balance is properly levelled and adjusted the following conditions must be observed:—

(1) The pointer should be opposite to the middle division of the scale, the beam at the same time being perfectly horizontal.

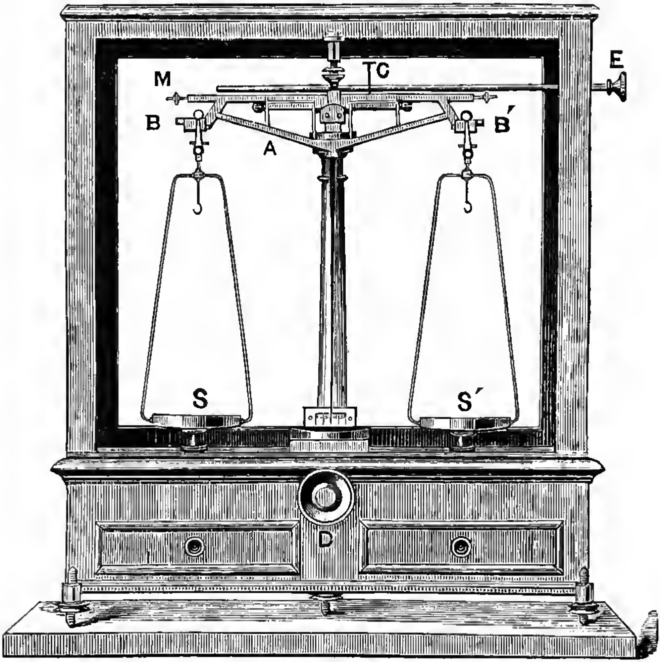


Fig. 21.

(2) The arms of the balance must be symmetrical, uniform, and of equal length, that is to say, the knife-edge on which the beam turns must be exactly midway between the knife-edges on which the scale pans hang.

(3) The scale pans must be of equal weight.

(4) The centre of gravity of the beam must be vertically *below* the axis of rotation when the beam is horizontal, as indicated by the pointer and scale.

The practical manipulation necessary to obtain the above conditions will be better acquired by the student with a balance before him and a few words of instruction from his teacher than from a long description accompanied by figures or cuts.

For a general theory of the balance with straight beam, see Appendix, § 5. The smaller weights accompanying the balance are usually marked thus:—

(1)	0·5 to 0·1	of a gramme or	decigrammes.
(2)	0·05 to 0·01	„	centigrammes.
(3)	0·005 to 0·001	„	milligrammes.

For fine weighing a wire rider 1 milligramme in weight can be placed on the balance beam, the arm of which is divided into ten equal parts. By putting the rider, using the rod E and hook C, on the first division from the centre of the beam it will counterpoise one-tenth of a milligramme on the opposite scale pan.

In order to weigh a body the position of the pointer when at rest must first be determined. If it does not rest at the zero of the scale, note its position and use that as the working zero. If the pointer be not quite at rest, note the numbers at which it turns on each side of the middle of the scale, and take the mean as the zero; for greater exactness another pair of observations may be taken as the swings grow less. Be careful always to support the pans by the rests, when the body or weights

are put in or taken from the pans; and never exceed the limited load which the balance was made to weigh.

By trial and error find the weights which equipoise the body, *i.e.* when the pointer rests at its working zero. As the pointer will take some time in coming to rest, note the turning-point on each side of the working zero, and add a small weight (or use the rider) to the side at which the swing is greatest, until the pointer vibrates equally on each side of its zero. The true weight can also be found, without the last adjustment, by observing the oscillation of the pointer, as follows:—

Note the limits of its swing and estimate the point of rest (a), then add a milligramme or two so that the pointer swings on the other side of its zero; again note the limits of the swing and estimate the new point of rest (b). The difference of the two points of rest gives the deviation (c) for the small weight added. The total weight in the pan *plus* the small weight added was therefore too heavy by the difference between (b) and the true zero divided by the deviation (c).

Example.—The scale reads from 0 to 50, division 25 being the true zero.

If point (a) = 29, and with *one* milligramme the point (b) = 18,

$$\therefore c = 29 - 18 = 11 \text{ divisions,}$$

therefore $\dfrac{25-18}{11} = \dfrac{7}{11}$ of a milligramme to be taken from the weight in the pan.

The number of scale divisions between the two estimated points of rest (a) and (b) enables us to

determine the *sensitiveness* of the balance. Let the small weight added be W, and the points of rest be a and b, then $\frac{a-b}{W}$ will be the sensitiveness of the balance for the particular load on the pans.

Example.—If the pointer stands at 25 when 100 grammes are on each pan, and at 16 when the difference is ·001 of a gramme, or one milligramme, therefore the sensitiveness is $=\frac{25-16}{1}=9$ divisions per milligramme for a load of 100 grammes.*

If the arms of the balance are not quite of equal length, we may determine the correct weight of a body by *double weighing*, that is by weighing with the body in one pan, then changing the body into the other pan and weighing again.

Let a and b be the lengths of the left and right arms of the balance respectively,

W = true weight of the body,

W_1 = apparent weight when the body is in the right-hand pan,

W_2 = apparent weight when in the left-hand pan.

Then

(1) $W_1 a = Wb$ in the first weighing

(2) $Wa = W_2 b$,, second ,,

* Theoretically the sensitiveness is the same whatever the load, but owing to the bending of the beam and other causes the sensitiveness varies slightly with the load.

Dividing equation (1) by (2)

$$\frac{W_1}{W} = \frac{W}{W_2},$$

$$\therefore W = \sqrt{W_1 W_2}.$$

Example.—The apparent weights of a body when weighed in the left and right-hand pans are 26·245 and 26·217 grammes respectively, therefore the true weight is

$$W = \sqrt{26{\cdot}245 \times 26{\cdot}217} = 26{\cdot}231 \text{ grammes.}$$

Exercises.

1. Find the true weight of a body in a balance by the method of double weighing.

2. Find the sensitiveness of the balance for loads of different amounts, and represent the results graphically with loads as abscissæ, and sensitiveness as ordinates.

Specific Gravity.

The specific gravity of a body is the ratio of its density (p. 9) to that of a standard substance, usually water at 4° C. It is therefore a numerical quantity, and has the same value whatever units are employed. Density and specific gravity coincide in numerical value when the density of the standard substance is taken as unity. The specific gravity of a substance being its weight divided by the weight of an equal volume of water at 4° C., the latter is conveniently found by noting *the loss of weight* the body undergoes when weighed in water; as the temperature of a room is about 15° C., a correction becomes

necessary for the slight difference in the density of water, and also for the buoyancy of the air (see p. 81).

Experiment 23.—**To find the specific gravity of an insoluble body *heavier* than water.**

Instruments required.—A balance, a box of gramme weights, a beaker of distilled water, and a thermometer.

When the chemical balance is used for the determination of specific gravities, there is usually a hook fixed immediately below the knife-edge, at the end of the arm, to which the body is hung by a fine thread when it is being weighed in a liquid.

There is also a movable platform, which is made to rest on the floor of the balance case, and overspanning the scale pan, thus supporting the beaker of liquid in which the body is immersed. If, owing to the mode of suspension or the smallness of the scale pan, a platform of this kind cannot be used, the pan is taken off the balance. and a counterpoise with suitable hook put in its place.

To make the experiment, first weigh the body in air in the usual way, then suspend it by a fine thread from the hook, having the body *completely* immersed in the water and free from all air bubbles, which latter may be detached by the feather end of a quill (see also p. 82). Then if

W = weight in grammes of the body in *air*,
W_1 = " " " *water*,
ρ = the specific gravity of the body,

$$\rho = \frac{W}{W - W_1}.$$

Example.—Find the specific gravity of a piece of brass.

$$\rho = \frac{15\cdot9}{15\cdot9 - 14} = 8\cdot03.$$

Exercises.

1. Find the specific gravity of a glass stopper.
2. Find the specific gravity of a piece of basalt.

Experiment 24.—**To determine the specific gravity of an insoluble body *lighter* than water.**

Instruments required.—The same as before, together with a suitable sinker, which may be made of a piece of brass with a hook for suspension on one side and a sharp spike to thrust in the light body on the other; this saves tying on the sinker to the light body.

Proceed with the weighing as in the last experiment. Then if

W = weight of the *solid* in *air*,

W_1 = weight of the *sinker* in *water*,*

W_2 = weight of the *solid* and *sinker* together in water,

ρ = the specific gravity of the body,

$$\rho = \frac{W}{W + W_1 - W_2} \quad \text{(Appendix, § 6).}$$

Example.—Find the specific gravity of a piece of paraffin wax.

$$\rho = \frac{7\cdot9}{7\cdot9 + 14\cdot05 - 12\cdot9} = 0\cdot87.$$

Exercise.

Find the specific gravity of a piece of cork.

* It is not necessary to weigh the sinker in air, as this weight cancels out.

Experiment 25.—**To determine the specific gravity of a *soluble* body heavier than water.**

Instruments required.—Same as in Experiment 23, and a liquid lighter than the body, and of density ρ_2, in which the body is not soluble.

Proceed as before, then if

W = weight of the body in air,

W_1 = weight of the body in the liquid,

ρ_1 = specific gravity of the body compared with the liquid,

ρ = specific gravity of the body compared with water,

$$\rho_1 = \frac{W}{W - W_1},$$

and $$\rho = \rho_1 \times \rho_2.$$

Example.—Find the specific gravity of a piece of lump-sugar, the liquid used being petroleum with $\rho_2 = 0{\cdot}83$.

$$\rho_1 = \frac{6{\cdot}6}{6{\cdot}6 - 3{\cdot}19} = 1{\cdot}94,$$

$$\rho = 1{\cdot}94 \times {\cdot}83 = 1{\cdot}61.$$

Exercise.

Find the specific gravity of a piece of rock-salt.

Experiment 26.—**To determine the specific gravity of a liquid: method i.**

Instruments required.—Same as in Experiment 23, also a body for displacement of the liquid, which body

must be heavier than, and not acted on chemically by, either the liquid or water.

Proceed as before, then if

W = weight of the body in air,
W_1 = „ „ „ water at $t°$ C.,
W_2 = „ „ „ the liquid also at $t°$ C.,
ρ = the specific gravity of the liquid,

$$\rho = \frac{W - W_2}{W - W_1}.$$

Example.—Find the specific gravity of petroleum at 15° C., using a glass stopper as the body for displacement.

$$\rho = \frac{15 \cdot 9 - 14 \cdot 37}{15 \cdot 9 - 14 \cdot 05} = 0 \cdot 827.$$

Exercise.

Find by this method the specific gravity of a solution of zinc sulphate.

Experiment 27.—**Determination of the specific gravity of a liquid: method ii.**

Instrument required.—A Mohr's balance.

Mohr's balance (Fig. 22) consists of a beam ABC pivoted at B, and having on the longer arm BC nine notches in which riders can be placed. A float F is hung from a hook C by means of a fine platinum wire. This float is conveniently made of a small thermometer, which at the same time gives the temperature of the liquid.

There are three riders, having a weight ratio to each

other of 1, $\frac{1}{10}$, $\frac{1}{100}$, accompanying the instrument, and when the rider 1 is hung on the hook C,* the float being

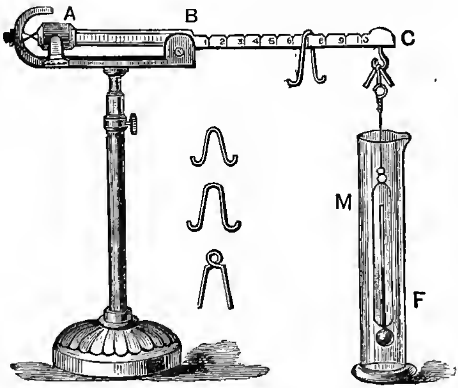


Fig. 22.

wholly immersed in distilled water at 15° C., the instrument is in equilibrium, indicated by the two pointers at A being in a line.

The liquid the density of which has to be determined is contained in the vessel M with the float immersed in it; riders are added to the beam till a balance is obtained.

Example 1.—Find the density of spirits of wine.

First rider was at 8. Second rider was at 2. Third rider (hanging on the first) was at 8.

$$\therefore \text{ density} = 0{\cdot}828.$$

Example 2.—Find the density of a solution of zinc sulphate.

As this liquid is heavier than water, a *fourth* rider

* The hook at C is under the 10th notch, and not as shown in the figure.

equal in weight to the *first* was put on the hook C, and the other three riders were at the positions 1, 2, 8.

$\therefore$ the density = 1·128.

Both examples require correcting for temperature (p. 81).

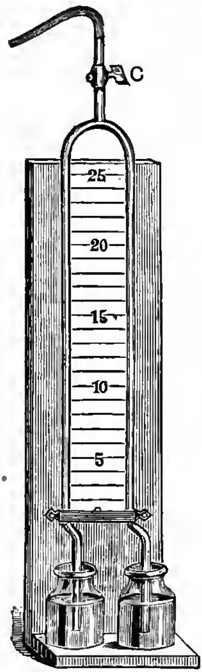


Fig. 23.

Exercise.

Find by Mohr's balance the specific gravity of

1. Pure alcohol.
2. Fusel oil.
3. Copper sulphate solution.

Experiment 28.—**To determine the specific gravity of a liquid: method iii.**

Instruments required.—A U-tube and a cathetometer or a scale.

(1) Take a tube (Fig. 23) with a stop-cock C at the bend, and the ends dipping into two vessels A and B containing water in the one, and the liquid, whose specific gravity is required, in the other.

When the tube is placed in position and some air drawn out by means of the stop-cock C, the liquids rise in their respective tubes, and the heights above the level of the liquids in the vessels will be in the inverse ratio of the specific gravities of the liquids.* If

* The scale shown needs shifting till its zero is level with the liquids in A B; if one scale be used the liquid surfaces must be at one level.

h and ρ = the height and specific gravity of one liquid,
h' and ρ' = the height and specific gravity of the other liquid, then

$$\rho h = \rho' h'.$$

(2) A more accurate way is to take a double U-tube (Fig. 24) with a stop-cock C at the upper bend.*

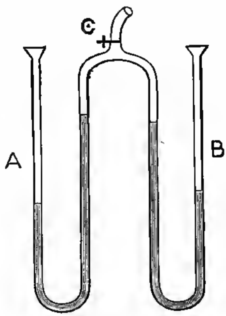


Fig. 24.

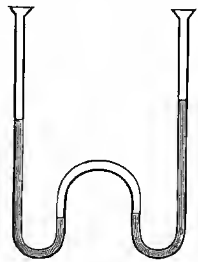


Fig. 25.

Pour the liquids whose specific gravities are to be compared, one into tube A and the other into tube B, the cock C being open. Then draw out some air and close C, when a difference of level will be established in each tube; finally measure this difference of level by means of the cathetometer as in the first case, and calculate as above.

Let d = difference of levels in the one tube and d' = difference of levels in the other tube, then

$$\rho d = \rho' d'.$$

* A much simpler apparatus (which has the advantage of avoiding any leak in the stop-cock or pinch-tap) can be made by the student for himself by bending a length of say four feet of glass tubing about half-inch diameter into the shape given in Fig. 25. By pouring the liquids alternately, a little at a time, each into its respective tube, the air entrapped in the bend will be compressed, and a difference of level in the liquids established as before. Wide tubing is used to avoid capillarity.

Example 1.—Find the specific gravity of petroleum, having water in the other limb of the U-tube.

By measurement with the cathetometer we have $h = 12{\cdot}6$ cms., $h' = 15{\cdot}2$ cms.

$$\therefore\ \rho' = \frac{1 \times 12{\cdot}6}{15{\cdot}2} = 0{\cdot}828.$$

Example 2.—Repeat Example 1 with the double U-tube. By measurement with the cathetometer the difference of levels of the water $= 6{\cdot}45$ cms., and the difference of the petroleum $= 7{\cdot}8$ cms.

$$\therefore\ \rho = \frac{1 \times 6{\cdot}45}{7{\cdot}8} = 0{\cdot}827.$$

Exercises.

1. Find the specific gravity of methylated spirit.
2. Find the specific gravity of glycerine.

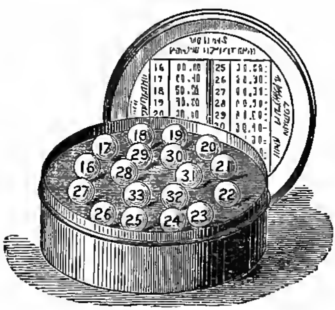


Fig. 26.

Experiment 29.—**To determine the specific gravity of a liquid by the specific gravity beads: method iv.**

Instruments required.—A vessel to contain the liquid under experiment, and a set of specific gravity beads.

Specific gravity beads are small hollow balls of glass (Fig. 26), each one marked with the specific gravity of the liquid in which it will just float.

To use the beads in determining the specific gravity of a liquid, take an assortment of beads having different values, and drop them one by one into the liquid till one of them just floats in the body of the liquid, *i.e.* has no tendency to either *rise* or *sink*, then the number marked on that bead is the specific gravity of the liquid. An ingenious modification of the specific gravity beads is shown in Fig. 27; the tube can be dipped in a liquid whose specific gravity is required, and the bead which floats gives the required density. The beads or discs are coloured differently, so that they can be readily distinguished.*

Example.—Find the specific gravity of dilute sulphuric acid by the *beads.*

By trial we find that the bead marked 1·015 *sinks*, and that the one marked 1·003 *floats*, whilst that marked 1·012 just floats, therefore the specific gravity of the liquid is = 1·012.

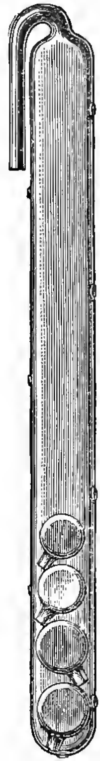


Fig. 27.

Exercise.

Find the specific gravity of olive oil.

A modification of the foregoing method † enables a rapid

* This arrangement is due to, and different modifications of it are made by, the well-known instrument maker, Mr. Hicks, of Hatton Garden, London.

† Professor Sollas in *Nature*, 26th February 1891.

determination to be made of the specific gravity of small fragments of a mineral, or of any minute insoluble solid object, up to a density of 3·45. In the case of bodies heavier than water, a heavy liquid such as methylene iodide is diluted with benzol, or potassium mercuric iodide is diluted with water, until a density is reached approximate to that of the body to be tested. It is best partly to fill a test-tube with the heavy liquid, pour on it the diluent, and leave it undisturbed till next day. A column of liquid of regularly increasing density from the top downwards will thus be obtained. Small specific gravity beads, of slightly different densities, are now thrown in, and when three or four sink to different levels their distance apart will be found to be directly proportioned to their respective densities. The distances and densities are now plotted on millimetre paper, and the body whose specific gravity is to be determined is dropped into the liquid, the exact position at which it floats is noted, and by reference to the curve already made its density is at once determined. Microscopic specimens can thus be examined in small tubes. The specific gravity beads can be readily made from bits of capillary glass tubing sealed and one end enlarged by blowing; the density of a number of these is determined beforehand by comparison with bodies of known density. The specific gravity of drops of aqueous liquids can also be found by this method, using methylene iodide diluted with benzol; for oily liquids a dilution of cadmium-boro-tungstate with water may be used. See Appendix, § 6 (3).

Experiment 30.—**Determination of the specific gravity of liquids by means of various hydrometers: method v.**

Instruments required.—Various hydrometers.

(i.) The ordinary hydrometer (Fig. 28) is an instrument of variable immersion, and consists of a glass tube with a bulb at the lower end, containing mercury to make it in stable equilibrium when floating in the liquid under experiment.

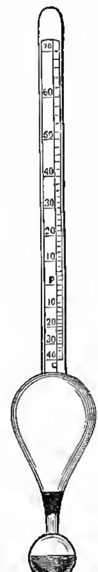


Fig. 28.

The tube or stem is graduated so that the instrument measures through a definite range of density. When immersed in the liquid under trial, the density of the liquid is shown by the depth to which the instrument sinks, the mark on the stem, level with the surface of the liquid, being read off (Appendix, § 6 (2)).

(ii.) In the case of Twaddell's hydrometer, instead of one instrument with a very long stem being employed to cover the range of the specific gravities of ordinary liquids, several short-stemmed instruments are made with mercury in each, so adjusted that the range of one ends where the next one *begins.*

A formula is given for use with this form of ordinary hydrometer. Thus for Twaddell's hydrometer the rule is, "Multiply the reading on the instrument by 0·005 and add 1."

(iii.) *Sike's hydrometer* (Fig. 29) is the form adopted by the Inland Revenue Department for determining the percentage of alcohol in spirits. It is made of gilt

brass and consists of a bulb with stem divided into ten parts or degrees; at the end of a lower submerged stem is a collar for the purpose of supporting a series of movable weights contained in the box.

The instrument floats at 0 on the stem without weights in strong spirits of density 0·825, in weaker

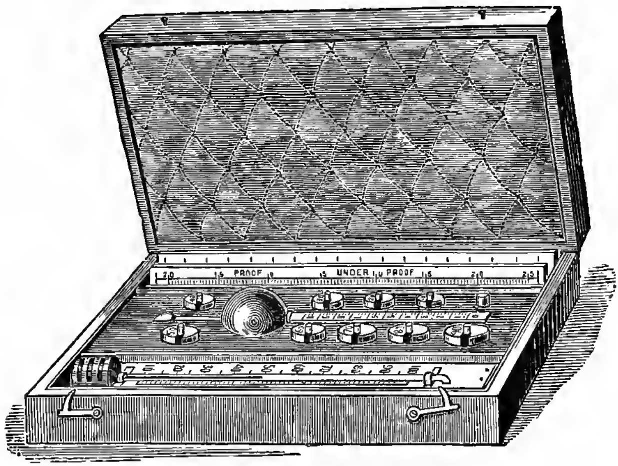


Fig. 29.

spirits weights are added to make it sink; and with the weight marked 90 (corresponding to 90 divisions of the scale) the instrument floats at 10 in distilled water; there is therefore a range of 100 degrees (*i.e.* 90 + 10 on the stem) between 0·825 and 1·000, each degree therefore corresponds to 0·00175 variation of density.

Example.—Find the specific gravity of a solution of copper sulphate by means of Twaddell's hydrometer.

The reading on the instrument = 38,

$$\therefore \text{density} = 1 + 38 \times 0{\cdot}005 = 1{\cdot}19.$$

Exercises.

1. Find by means of Twaddell's hydrometer the density of a mixture of sulphuric acid and water.

2. Find by Sike's hydrometer the percentage of alcohol in the sample given you.

(iv.) *Nicholson's hydrometer* (Fig. 30) is a *constant immersion* hydrometer, and consists of a hollow metallic tube, having a scale pan at the upper end A, and at the lower end B a metallic basket with movable perforated lid, in which is put the solid body when being weighed in water. A bead C on the stem is the standard mark to which the instrument must be sunk at each observation.*

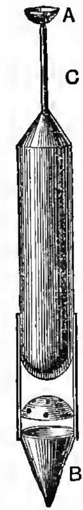


Fig. 30.

(1) To find the specific gravity of an insoluble body *heavier* than water. First, when the hydrometer is immersed in water, find what weight (W) put on the upper pan will sink it to the standard mark; this is the standard weight, then put the body on the upper pan together with weights so as to sink the instrument to the standard mark (the body under trial must always weigh less than the standard weight of the instrument). Then put the body in the lower pan and weigh again. Then if

W = standard weight,

W_1 = weight put on the upper pan along with the body to sink the instrument to the standard mark,

* To prevent the instrument sinking too far through overloading, a collar of wood or stiff card, with a slot for the stem, should *always* be

W_2 = the weight in the upper pan, when the body is in the lower pan, required to sink the instrument again to the standard mark;

then $W - W_1$ = weight of the body in air,

$W_2 - W_1$ = apparent loss of weight of the body in water,

ρ = specific gravity of the body,

$$\therefore \rho = \frac{W - W_1}{W_2 - W_1}.$$

For an insoluble body *lighter* than water the formula requires to be slightly changed (see Example 2 below).

Example 1.—Find the specific gravity of a piece of bismuth.

$$\rho = \frac{8{\cdot}9 - 2{\cdot}03}{2{\cdot}73 - 2{\cdot}03} = 9{\cdot}81.$$

Example 2.—Find the specific gravity of a piece of mahogany.

$$\rho = \frac{W - W_1}{W_1 - W_2 + W}$$

$$= \frac{8{\cdot}9 - 6{\cdot}07}{6{\cdot}07 - 10{\cdot}65 + 8{\cdot}9} = 0{\cdot}655.$$

The advantage of a Nicholson's hydrometer is that an ordinary balance is not required with it, unless it be employed to determine the specific gravity of a liquid, when the weight of the hydrometer must be found first.

(2) To find the specific gravity of a liquid *heavier* than water by Nicholson's hydrometer.

placed on the top of the hydrometer jar. The water used should have been boiled, and air-bubbles adhering to the hydrometer must be removed.

Let

W = weight of the hydrometer,

W_1 = weight on the top pan when the hydrometer is in water,

W_2 = weight on the top pan when the hydrometer is in the liquid whose specific gravity is required,

ρ = specific gravity of the liquid,

$$\rho = \frac{W + W_2}{W + W_1}.$$

Example 3. Find the specific gravity of a solution of sulphate of copper. The hydrometer weighs 10·35 grammes.

$$\rho = \frac{10{\cdot}35 + 9}{10{\cdot}35 + 5{\cdot}77} = 1{\cdot}2.$$

Exercises.

1. Find the specific gravity of a piece of brass.
2. Find the specific gravity of a piece of cork.

Experiment 31.—**To determine the specific gravity of a liquid by the specific gravity bottle: method vi.**

Instruments required.—A balance, a box of gramme weights, together with a specific gravity bottle.

The specific gravity bottle is a bottle made of thin glass, with a ground stopper having a capillary bore; or one with an ordinary stopper, having simply a mark on the narrow neck of the bottle. The bottle when quite full up to the top of the stopper, or to the level of the mark, is made to contain a certain even number of grammes of

water, such as 50 or 100 grammes, at standard temperature. In place of a bottle a small beaker with ground lip and a ground glass lid may be used with advantage (especially for the determination of the specific gravity of fragments of mineral). The beaker is filled to overflowing with the liquid to be tested, and the ground lid slipped carefully over the top, so as to exclude all air bubbles. The beaker is then wiped clean and dry, and weighed.

Sprengel's sp. gr. tube for liquids is an instrument easily made by the student, and affords the simplest and most accurate method of determining the sp. gr. of liquids at any given temperature. It consists of a piece of glass tubing, say 6 mm. diameter (A A, Fig. 31), bent as shown, the ends drawn to a capillary bore, and it is convenient to have one arm a finer bore than the other. The liquid is drawn into the tube by suction, and its temperature can be rapidly brought to the required point by placing the tube in a beaker of water B, as shown; a mark *m* is made on the wider capillary, and up to this mark the capacity of the tube is once for all determined by weighing it empty and then full of water at the standard temperature.* It is necessary to let the liquid stand at the mark *m*, on the wider capillary bend, so that it can expand without loss on weighing. Then if

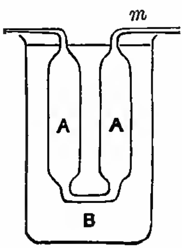


Fig. 31.

* The tube is weighed by suspending it from the hook of the balance by a thread slipped over the bent arms of the tube.

x = the number of grammes of water at standard temperature that the bottle, tube, or beaker contains,

W = the weight of the empty bottle or beaker,

W′ = the weight of the bottle or beaker when full of liquid,

ρ = specific gravity of the liquid,

$$\rho = \frac{W' - W}{x}.$$

Example.—Find the specific gravity of petroleum, using a 50 gramme bottle.

$$\rho = \frac{81 \cdot 9 - 40 \cdot 6}{50} = 0 \cdot 826.$$

Exercise.

Find the specific gravity of glycerine, also of copper sulphate solution.

Experiment 32.—**Determination of the specific gravity of a powder or small fragments of mineral by the sp. gr. bottle.**

Instruments required.—Same as in Experiment 31.

To find the specific gravity of a *powder* by the bottle or beaker. Find the weight of water held by the bottle at the temperature of the air. Weigh the powder to be used and pour it into the clean empty bottle, fill up the bottle with distilled water at the temperature of the air. *Carefully dislodge all air-bubbles held by the solid* either by

shaking or by the air-pump. Note the temperature. Let

x = weight of water which the bottle or beaker contains at standard temperature,

W = weight of the powder in air,

W_1 = weight of the *powder* and *water* in the bottle or beaker when quite full of water,

ρ = specific gravity of the powder.

$$\rho = \frac{W}{x + W - W_1}.$$

Example.—Find the specific gravity of a sample of crushed glass, using a 50 gramme bottle.

$$\rho = \frac{7{\cdot}64}{50 + 7{\cdot}64 - 54{\cdot}61} = 2{\cdot}52.$$

Exercise.

Find the specific gravity of a sample of sand.

Mr. Moss has modified Sprengel's sp. gr. tube and made it suitable for minute specimens, such as precious stones, as

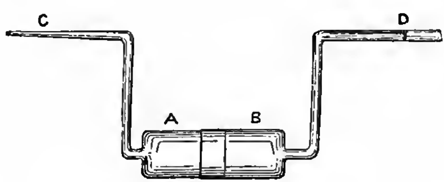


Fig. 32.

shown in Fig. 32. Two small pieces of glass tube, A B, are ground into one another, which can readily be done with

fine emery powder and water. The other ends of the tube are then drawn off to a capillary bore (one wider than the other, for the reason already given) and bent twice at right angles, as at C and D. The weight and capacity of the empty tube being determined once for all, the tube is taken apart and the weighed specimen, whose specific gravity is to be determined, is placed within A B; the tube being closed, is again filled with water and carefully weighed. The specific gravity is then calculated as in the previous experiment. To keep the temperature of the water in A B constant, or reduce it to 4° C., the tube can be placed in a beaker of water, the arms C D affording a convenient support. In refitting the tube the same pressure must be used, or its volume may alter.

Another method of determining the specific gravity of minute fragments is described in Appendix, § 6 (3).

Corrections and Precautions.

(i.) Since the specific gravity of a body is the weight of the body divided by the weight of an equal volume of water at 4° C., it is necessary to make a correction for the *difference in temperature* of the water above 4° C. This is easily done by reference to the tables of the density of water at different temperatures, and multiplying the specific gravity of the body as found by the specific gravity of the water at the temperature of the experiment, for the same reason as in weighing a body in a liquid other than water (see Table VIII.).

(ii.) As the weighings are *not made in vacuo* the apparent weight of the body in air is less than the true weight

by an amount equal to the weight of the air displaced (Experiment 36). Hence the *apparent specific gravity*, ρ, must be corrected for this and also for the temperature and different displacement at that temperature of the equivalent bulk of water. *The true specific gravity*, corrected for both temperature and for displaced air, is sufficiently given by the formula $\rho (d - \delta) + \delta$, where d is the density of water at the temperature of the room, and δ the density of air compared with water, which may be taken as 0·0012. It will be seen that the correction for buoyancy of the air vanishes when the density of the body is 1, and becomes larger the more the density differs from 1.

(iii.) The foregoing corrections also apply to the results obtained by the specific gravity bottle, and in this case when greater accuracy is required a correction must be made for the expansion of the glass. This correction may become important in using a Nicholson's hydrometer, as the volume will alter with the temperature.

(iv.) In weighing a body in water the submerged portion of the thread or wire which suspends the body loses weight, and in accurate work this must be allowed for by noting the length submerged, the weight and specific gravity of the wire being known.

(v.) The removal of all adhering bubbles is obviously most important. If shaking and brushing the object fail, the water should be heated or an air-pump used; especial care is needed in this direction when the specific gravity of a powder is determined by the specific gravity bottle. Distilled water freed from air by boiling should be used. The specific gravity bottle should be carefully cleaned and

dried. The latter may be accomplished by warming it and blowing in air through a narrow tube from a foot-bellows. Rinsing with alcohol hastens matters.

Experiment 33.—**To determine the volume and density of a solid by the stereometer.**

Instrument required.—A stereometer.

A stereometer, or as it is sometimes called a volumenometer, is an apparatus for determining the volume of solids which are affected by water, or the density of which cannot be obtained in the ordinary way: such, for example, as wool, cotton, gunpowder, etc. The instrument is an application of Boyle's law (see Experiment 43), the mass in grammes divided by the volume in cubic cms. giving the density of the body.

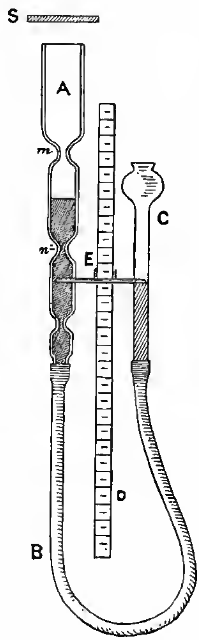


Fig. 33.

There are several forms of stereometer, but the modification shown in Fig. 33 is simple and effective. The substance whose volume is required is placed in a perfectly dry glass tube A, which is contracted in two or three places. On the ground top of A a ground glass cover S can be securely fastened by a screw; a flexible tube B connects A with a second glass tube C. Clean dry mercury is poured into C and allowed to rise in A by lifting the tube C until it reaches a fine mark *m*.

The greased cover is then fastened down, and the tube C lowered until the mercury stands at a mark n in the lower neck of A. Fix the tube C and read the level of the mercury in A and C; this can be done by the scale D, or by a cathetometer. The volume v between the marks m and n can now be calculated if we know the volume V of the receiver above the mark m. This can be found by putting a body of known volume, such as a piece of glass rod, into the receiver, closing the lid when the mercury is at m, and proceeding as before. The volumes of V and v are, however, best found directly by allowing mercury to run into A from a burette, the rubber tube B meanwhile being closed by a strong pinch-tap, nipped close to A to avoid extension. Greater accuracy can be obtained by running off the mercury in A and weighing it, but this necessitates a stop-cock or two.

The solid whose volume is required is now weighed and introduced into the receiver A; the quantity of the solid taken should be large enough nearly to fill the receiver. When the mercury stands at m, the greased lid is closed and the tube C lowered as before until the mercury stands at n; the difference of level between the mercury in the tubes A and C is now accurately read. This enables us to find the volume of the receiver *minus* that of the solid; since the former is known the latter is at once obtained.

Let V = volume of receiver,

v = volume of tube between marks m and n,

H = barometric height at the time of experiment,

h = difference of level of mercury in A and C when the body is *not* in the receiver,

$h' =$ difference of level of mercury in A and C when the body is in the receiver,

$x =$ volume of body to be found.

Then, by Boyle's law, to find v,

$$V : V + v = H - h : H.$$

And to find x,

$$V - x : (V + v) - x = H - h' : H.$$

Example 1.—Find the volume v.

$$V = 53{\cdot}5 \text{ c.c.},$$
$$H = 76{\cdot}4 \text{ cms.},$$
$$h = 14{\cdot}0 \text{ cms.},$$

then $\quad 53{\cdot}5 : 53{\cdot}5 + v = 76{\cdot}4 - 14{\cdot}0 : 76{\cdot}4;$

hence $\quad v = 12{\cdot}0$ c.c.

By direct measurement v was found to be $12{\cdot}1$ c.c.

Example 2.—To find x, the volume of a sample of gunpowder.

$$V = 53{\cdot}5 \text{ c.c.},$$
$$v = 12{\cdot}1 \text{ c.c.},$$
$$H = 76{\cdot}4 \text{ cms.},$$
$$h' = 18{\cdot}7 \text{ cms.},$$

then $\quad 53{\cdot}5 - x : 65{\cdot}6 - x = 57{\cdot}7 : 76{\cdot}4;$

hence $\quad x = 16{\cdot}2$ c.c.

The weight w of the gunpowder was $13{\cdot}77$ grammes, hence

$$\rho = \frac{w}{x} = \frac{13{\cdot}77}{16{\cdot}2} = 0{\cdot}85.$$

Exercise.

Find the density of cotton-wool by the stereometer.

Experiment 34.—**To determine the weights of constituents in a mechanical compound, when the specific gravities of the constituents and compound are known.**

Instruments required.—The same as in Experiment 23.

If there are *two* constituents in the compound first determine the specific gravity of the compound body in the usual way, then if W, W_1, W_2 be the weights of the compound body and the two constituents respectively, and ρ, ρ_1, ρ_2 their respective specific gravities, then

$$W_1 = \frac{(\rho_2 - \rho)\rho_1}{(\rho_2 - \rho_1)\rho} W,$$

$$W_2 = \frac{(\rho_1 - \rho)\rho_2}{(\rho_1 - \rho_2)\rho} W.$$

Example.—A brass tube of specific gravity $\rho_1 = 7{\cdot}6$ is filled with lead of specific gravity 11·3. To find the weights of the brass and lead. The weight of the whole in air is 34·12 grammes, and in water 30·9 grammes. By the ordinary method

$$\rho = \frac{34{\cdot}12}{34{\cdot}12 - 30{\cdot}9} = 10{\cdot}6,$$

$$W_1 = \frac{(11{\cdot}3 - 10{\cdot}6)7{\cdot}6}{(11{\cdot}3 - 7{\cdot}6)10{\cdot}6} 34{\cdot}12 = 4{\cdot}629 \text{ grammes},$$

$$W_2 = \frac{(7{\cdot}6 - 10{\cdot}6)11{\cdot}3}{(7{\cdot}6 - 11{\cdot}3)10{\cdot}6} 34{\cdot}12 = 29{\cdot}49 \text{ grammes}.$$

Exercises.

1. Find the relative weights of gold of specific gravity 19·3; and quartz of specific gravity 2·65 in a nugget.

2. Find the weight of a bullet of specific gravity 11·3 buried in a mass of wax of specific gravity 0·87.

Experiment 35.—**To determine the sp. gr. of a mixture of two liquids, which contract on mixing, the volumes and sp. gr. of the liquids being known.**

Instruments required.—A burette and a graduated tube.

Measure the volumes of the liquids taken by means of the burette, then run them into the graduated measuring glass and observe the contraction.

Let v and v_1 = volume of the liquids,

ρ and ρ_1 = sp. gr. of the liquids,

ρ_2 = sp. gr. of the mixture.

Let $v + v_1$, when mixed, be reduced $1/nth$, as found by experiment. Then

$$\rho_2 = \frac{n}{n-1} \times \frac{v\rho + v_1\rho_1}{v + v_1},$$

and if the specific gravity of the mixture be known, to find the contraction

$$\frac{1}{n} = 1 - \frac{v\rho + v_1\rho_1}{(v + v_1)\rho_2}.$$

Example 1.—Find the specific gravity of a mixture of equal parts of water and alcohol at temperature 15° C.

Here
$$v = v_1 = 22 \text{ c.c.},$$
$$\rho = 0{\cdot}9998, \quad \text{and } \rho_1 = 0{\cdot}7995,$$

and
$$n = \frac{50}{2{\cdot}1} = 23{\cdot}81,$$

$$\therefore \rho_2 = \frac{23{\cdot}81}{22{\cdot}81} \times \frac{25 \times {\cdot}9998 + 25 \times {\cdot}7995}{50}$$

$$= \frac{23{\cdot}81 \times 44{\cdot}98}{22{\cdot}81 \times 50} = 0{\cdot}938.$$

Example 2.—If the specific gravity of the above mixture has been found to be 0·939, then

$$\frac{1}{n} = 1 - \frac{25 \times \cdot 9998 + 25 \times \cdot 7995}{50 \times \cdot 939}$$

$$= \frac{44 \cdot 95}{50 \times \cdot 939} = \cdot 0428 = \frac{2 \cdot 14}{50},$$

or the contraction is about $\frac{1}{25}$ of the whole volume.

Exercises.

1. Mix 30 c.c. of water with 45 c.c. of alcohol, and measure the contraction and specific gravity, correcting for temperature (see Table VIII.).

2. Mix 10 c.c. of sulphuric acid of specific gravity 1·84 with 50 c.c. of water. Note the temperature *before* and *after* mixture, and when the mixture has cooled to the temperature of the air, find its specific gravity, and estimate the contraction.

Experiment 36.—**To determine the *true* weight of a body—*i.e.* its weight in vacuo.**

Instruments required.—The same as in Experiment 23.

Proceed as in that experiment. If

W_1 = weight of the body in air at $t°$ C.,

W_2 = weight of the body in water at $t°$ C.,

W = the required weight in vacuo,

ρ = the specific gravity of air compared with water at $t°$ C.,

then (see p. 66)

$$\rho = \frac{W - W_1}{W - W_2},$$

$$\therefore\ W = \frac{W_1 - \rho W_2}{1 - \rho}.$$

Example.—Find the *true* weight of an ivory ball.

$$W_1 = 73{\cdot}447 \text{ grammes},$$
$$W_2 = 34{\cdot}248 \text{ grammes},$$
$$\rho = 0{\cdot}0012 \text{ (from Table VI.)},$$
$$\therefore\ W = \frac{73{\cdot}447 - (0{\cdot}0012 \times 34{\cdot}248)}{1 - 0{\cdot}0012}$$
$$= 73{\cdot}51 \text{ grammes}.$$

Exercises.

1. Find the true weight of a piece of wood (coat the wood with a thin layer of shellac varnish to prevent the absorption of water).

2. Find the true sp. gr. (p. 82) of a platinum crucible, correcting for the temperature of the air and water.

Experiment 37.—**To determine the density of a gas.**

Instruments required.—A balance and a light glass globe with stop-cock.

The globe is first exhausted of all air and carefully weighed, it is then filled with dry air at the temperature and pressure of the surrounding atmosphere and weighed

again. The globe is now carefully exhausted by means of the air-pump, and the gas to be tested allowed to flow in; the globe is once more exhausted and again filled with the same gas, so as to insure that it is free from traces of air. Care should be taken that the gas in the globe is at the temperature and pressure of the atmosphere, which is noted; the globe full of gas is now carefully weighed. Then the ratio of the weight of the gas to the weight of the air gives the density of the gas compared with air.

If the gas and the air are not at the same temperature and pressure in the two experiments, a reduction must be made.

The relative density of a gas may also be found by diffusion or effusion, as described in Chapter X.

The experimental determination of the absolute density of a gas requires great precautions, the description of which is beyond the scope of this book.

Example 1.—Find the weight of 1000 c.c. of dry air.

By immersion in water the volume of the globe used was found to be approximately 3060 c.c., and by weighing the globe empty and full of air, it was found to contain 3·95 grammes of air.

$$\therefore \; 1000 \text{ c.c.} = 1{\cdot}29 \text{ grammes.}$$

Example 2.—Find the relative density of oxygen gas, taking air as unity.

By experiment, using a different globe, it was found that—

$$\begin{array}{rlll} & 127{\cdot}290 \text{ grams.} & = \text{weight of globe full of air,} \\ & 125{\cdot}505 \quad ,, & = \quad ,, \qquad ,, \quad \text{empty,} \\ \hline \therefore & 1{\cdot}785 \quad ,, & = \text{weight of the air in globe.} \end{array}$$

$$\begin{array}{rlll} & 127{\cdot}476 \text{ grams.} & = \text{weight of globe full of oxygen,} \\ & 125{\cdot}505 \quad ,, & = \quad ,, \qquad ,, \quad \text{empty,} \\ \hline \therefore & 1{\cdot}971 \quad ,, & = \text{weight of the oxygen in globe.} \end{array}$$

$$\therefore \ \rho = \frac{1{\cdot}971}{1{\cdot}785} = 1{\cdot}104.$$

The temperature and pressure remained constant throughout the experiments.

Exercise.

Find the weight of 1000 c.c. of coal gas and its relative density compared with air.

Note.—In testing coal gas a globe with a stop-cock above and below may be used, one cock fixed to a gas bracket, the upper having a gas burner screwed to it. Both cocks being open the gas is turned on and allowed to stream through for a minute or two when the jet is lighted; after having burnt for, say, half an hour, the two cocks are turned off and the globe weighed. A similar proceeding may be adopted to fill it with dry air, which can be driven from a gas holder through drying tubes; temperature and pressure being the same in both cases, or corrected if different.

CHAPTER V

MEASUREMENT OF FLUID PRESSURE

The Barometer

THE characteristic property of gases is their power of indefinite expansion under diminished pressure. Boyle's law expresses the relationship between the pressure and the volume of a gas as follows: the volume (V) of any portion of a gas varies inversely as the pressure (P) to which it is subject, the temperature being constant, or $VP = V'P'$. The law of Charles expresses the relationship between the volume and the temperature of a gas, the pressure being constant, as follows: the volume of any given portion of gas is directly proportional to its absolute temperature (T) where $T = t°\ C. + 273$, or $VT' = V'T$. Combining the two laws, $VPT' = V'P'T$.

The atmospheric pressure (Π) is that pressure in dynes per square centimetre which is equivalent to the pressure of the mercury in a barometer tube. If H be the standard barometric height in cms. at 0° C., and "g" the force of gravity, which in Dublin $= 981{\cdot}3$, and ρ the density of mercury $= 13{\cdot}6$, then $\Pi = Hg\rho$ dynes per square centimetre, or $\Pi = 76 \times 981{\cdot}3 \times 13{\cdot}6 = 1{,}014{,}000$

dynes, that is a little over a megadyne (a million dynes) per square cm. It would be convenient, and it has been proposed to adopt exactly a megadyne per square cm. as the standard pressure instead of 76 cms. For Dublin this would mean a barometric height of 74·94 cms., or 29·5 in.

Experiment 38.—**To prove that the pressure at any point in water is proportional to the depth of the point below the surface.**

Instruments required.—A bent glass tube with one long and one short limb, and a millimetre scale.

Take a glass tube about 1 metre long and 0·5 cm. in diameter, make a bend at one end, the short limb being about 10 cms. long. Pour some mercury into the bend and it will stand at the same level, since the tube is open at both ends.

Now take a tall jar full of water and place the tube at various depths in the water, the long limb of the tube passing up through the surface of the water, and being open to the air.

Measure the difference of the level of the mercury in the two limbs at the various depths. This difference will be a measure of the pressure due to the depth of the end of the short limb below the surface of the water, the pressure of the air is the same on both limbs, and can therefore be neglected. Then if

d = depth of the end of the short limb below the surface,
h = difference in the levels of the mercury in the limbs.

$$\therefore \frac{d}{h} = \text{a constant.}$$

Example.—Proof of the above law.

Enter results thus:—

d in Cms.	h in Cms.	$\frac{d}{h}$
4	0·17	24·1
14	0·57	24·5
24	0·95	24·3
30	1·22	24·5
35	1·42	24·7
40	1·60	25·0
45	1·80	25·0
50	2·00	25·0
55	2·20	25·0

Exercises.

1. Make a U-tube and repeat the above experiment, plot your results in a curve on millimetre paper.

2. Make several tubes with the ends of the short limbs pointing in various directions, and show that the pressure in a fluid is the same in all directions.

The following experiment affords another proof of the foregoing law, but is one more suitable for lecture illustration than accurate measurement. Lower a glass cylinder open at both ends, such as a tall Argand lamp chimney, into a vessel of water; the lower end of the cylinder is ground and covered with a ground disc of metal or glass, to the centre of which is fixed a thread or

wire. Holding the disc by the thread or wire against the bottom of the cylinder, lower it to the bottom of a tall jar of water. Now release the thread and slowly raise the cylinder, at a certain depth below the surface of the water the disc falls off, measure this depth, weigh the disc and find the sectional area of the cylinder. The weight of the disc should be equal to the upward pressure of the water at the depth where the disc fell.

Example.—The disc fell at a depth of 10·5 centimetres from the surface of the water. Diameter of metal disc 7·4 cm., area = 43 square cms., weight of the disc 445 grammes. Hence as 1 c.c. of water weighs 1 gramme, the depth at which the disc ought to have fallen is $\frac{445}{43} = 10 \cdot 3$ centimetres below the surface. Experiment gave 10·5.

Exercise.

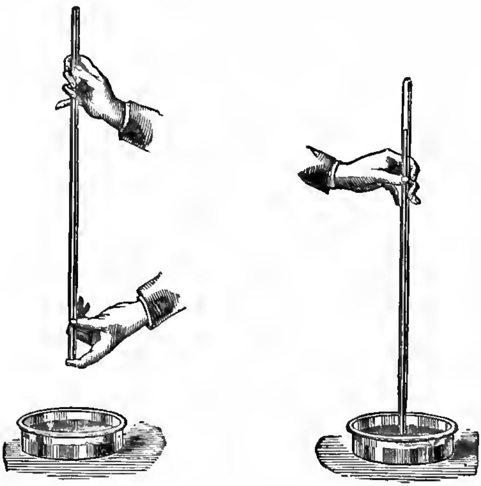
Repeat the foregoing experiment with the disc loaded; or in a liquid of greater density than water, and estimate the pressure corresponding to the depth at which the disc falls.

Experiment 39.—**To determine the pressure of the atmosphere by the barometer.**

Instruments required.—Barometer tube and mercury, also a standard barometer.

The construction of a barometer is shown in Fig. 34. A tube of glass about 1 metre long, closed at one end, is

nearly filled with pure warm mercury, the tube itself being dry and warm. By closing the open end with the finger the air bubbles attached to the sides of the glass can be removed by allowing a large bubble of air to travel up and down the tube, and afterwards tapping the



A Fig. 34. B

closed end of the tube on a soft surface. The tube is then completely filled with mercury, and inverted as shown in A, Fig. 34. Removing the finger the mercury falls until it stands at a height corresponding to the atmospheric pressure, as shown in B.

Fortin's standard barometer (Fig. 35) consists of a barometer tube dipping into a vessel of mercury A, the bottom of this vessel is made of wash-leather, which can be raised or lowered by means of the screw B, and so bring the surface of mercury to coincide with an ivory

point, which is the zero of the barometer scale. An enlarged view of the cistern is shown in Fig. 36. Instead of the ivory point a platinum pin may be fixed over the mercury in the cistern, with one end joined to a galvanometer and single cell, and a wire from the other pole of the cell to the mercury in the cistern. By pressing a key in the circuit the precise moment of contact between the mercury and platinum point is indicated by the galvanometer. When there is any difficulty in illuminating the ivory point, this method is convenient.

A brass tube, containing a scale near the top, encloses the glass tube of the barometer, and has at its upper end two vertical slots through which the top of the mercury column can be seen. In these slots a slider with vernier moves by means of a rack and pinion. In reading the height of the barometer the eye of the observer is placed *on a level* with the top of the mercury column, and the vernier moved until the light which passes between the top of the mercury and the vernier is just excluded.

If the eye be not level with the top of the mercury the reading will be too high, owing to the front and back edge

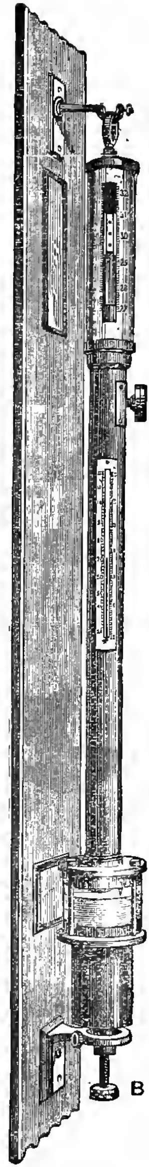


Fig. 35.

of the slider and the top of the mercury not being in one straight line. The height of the mercury is now read off by means of the scale and vernier. There are usually two scales on the standard barometer, a scale of inches, with "least count" on the vernier = ·002 inch, and a scale of cms. with "least count" = ·05 mm.

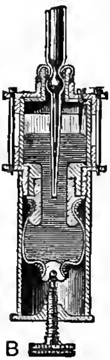


Fig. 36.

Correction for *capillarity*, which causes a depression of the mercury column, must be allowed for if the diameter of the tube be less than $\frac{3}{4}$ of an inch (see Table X.). The correction is, however, a little uncertain, and it is better to avoid it by using wide tubes.

A correction must also be made for the expansion by heat of the brass tube and of the mercury, when great accuracy is required.

Then if

H = barometric height in *inches* or *cms.* at 0° C.,
h_t = barometric height in *inches* or *cms.* at t° C.,
t = temperature of mercury and brass,

$$\mathrm{H} = h_t(1 - 0{\cdot}000162t).$$

(See Appendix, § 7).

In making an observation (i.) read the temperature of the attached thermometer before the observer's body has altered it; (ii.) adjust the level of the mercury in the cistern by the screw B until the ivory point coincides

with its image reflected from the mercury; (iii.) gently tap the barometer tube to overcome the "stiction" of the mercury; (iv.) set the vernier, avoiding parallax, and illuminating the background by a sheet of white paper; (v.) carefully read both verniers, using a lens if necessary.

Example.—Read the height of the barometer at 15° C., and reduce to 0° C.

Here $h_t = 29{\cdot}48$ inches,

$$\therefore \text{H} = 29{\cdot}48(1 - {\cdot}000162 \times 15) = 29{\cdot}41 \text{ inches.}$$

Exercise.

Read the barometric height in inches and millimetres, and reduce to 0° C.

Note.—To reduce the barometric reading to sea-level at latitude 45°.

Let

H_1 = height in metres of the place of observation above sea-level,

θ = latitude of the place,

h = observed barometric height,

h_0 = height corrected to sea-level;

then $h_0 = h(1 - {\cdot}0026 \cos 2\theta - {\cdot}0000002\text{H}_1)$.

Experiment 40.—**To determine the pressure of the atmosphere by simple means.**

Instruments required.—A narrow glass tube, pure mercury, and a millimetre scale.

Take a clean, dry, glass tube, say 40 cms. long and 3 mm. in diameter, closed at one end, heat the tube

gently over a Bunsen flame to expand the enclosed air, then plunge the open end into a beaker of pure mercury.

When it has cooled to the temperature of the air lift the tube up vertically and measure the length of the air column and the length of the mercury thread sustained in the tube by the atmospheric pressure, and which is prevented from falling out by capillarity. Now turn the tube upside down, and again measure the length of the air column.

In the first instance the enclosed air is dilated by the mercury column, and in the second instance compressed. Hence, by Boyle's law (see Experiments 42 and 43), if

H = the barometric height,
h = length of mercury in the tube,
l = length of air column in first case,
l' = length of air column in second case,

then
$$l(\mathrm{H}-h)=l'(\mathrm{H}+h);$$

$$\therefore\ \mathrm{H}=h\frac{l+l'}{l-l'}.$$

Example.—With a tube 35 cms. long and 3 mm. in diameter, closed at one end, we obtained

$$h=58{\cdot}2 \text{ mm.},\quad l=265 \text{ mm.},\quad l'=227 \text{ mm.}$$

$$\therefore\ \mathrm{H}=\frac{58{\cdot}2\times 492}{38}=753{\cdot}6 \text{ mm.}$$

The height of the standard barometer in the laboratory at the time of the experiment was 753·2 mm.

Exercise.

Repeat the above experiment, and compare the result with the standard barometer.*

Experiment 41.—**Determination of heights by the barometer.**

Instruments required.—A portable mercurial, or a delicate aneroid barometer.

To measure a small vertical height between two stations, read the barometer at the two places, and the difference in the readings gives the pressure of the column of air between the stations in millimetres or inches of mercury.

If the mercurial barometer is employed, care must be taken that the tube is vertical, which can be ensured by a plumb-line or by taking the lowest reading of the mercury column.

If the aneroid be used the instrument must be held in the same position at the two stations, that is to say either vertically at both places or horizontally.† Let

H = height required between the two places,

h = difference of the barometer or aneroid readings expressed in centimetres of mercury,

ρ = density of mercury at $t°$ C.,

δ = density of the air at $t°$ C.,

* Mr. Joly, of Trinity College, Dublin, by means of a plug of ivory or boxwood that nearly fills the tube, enables a glycerine or long-range barometer to be made of moderate height (see Appendix, § 8).

† A new and portable form of aneroid, devised by Captain Watkin and made by Messrs. Hicks, of Hatton Garden, London, enables a difference of level of 5 feet to be easily and accurately read.

then $h\rho =$ difference in height between the stations expressed in centimetres of a water barometer,

$\mathrm{H} = \frac{h\rho}{\delta} =$ height of the air column between the two stations which are not far apart.

Example.—Find the height from the bottom to the top step of a staircase. Temperature of air 12° C.

By using a Watkin's aneroid the readings at the two places were

At bottom	. .	29·938 inches
At top	. .	29·905 inches

$\therefore$ the difference $=$ ·033 inches $=$ ·084 cms.

and since $\rho = 13{\cdot}56$, and $\delta = {\cdot}001239$ (see Tables V. and IX.),

$$\therefore \mathrm{H} = \frac{{\cdot}084 \times 13{\cdot}56}{{\cdot}001239} = 919{\cdot}4 \text{ cms.} = 30{\cdot}16 \text{ feet.}$$

The height as measured by a tape was 30 feet 2 inches.

Exercise.

Find by the barometer the height from the basement to the top storey of a house or tower, and check your result if possible by direct measurement.

Note.—For an accurate determination, corrections must be made for the different density of the air at the observed height, and the hygrometric state of the air (see Maxwell's *Theory of Heat*, chap. xiv.). For the measurement of greater heights by the barometer, Laplace's formula, as modified by recent determinations of the density of mercury and of air, can be used (see Jamin and Bouty, *Cours de Physique*, vol. i. chap. iv.).

Experiment 42.—**To prove Boyle's law for pressures above that of the atmosphere.**

Instruments required.—A U-tube, a centimetre scale, and pure dry mercury.

In Fig. 37 ABC is the tube about 100 cms. long, open at C and closed at A. S and S′ are two centimetre scales for measuring the height of the mercury in the two limbs of the tube, which must be clean and dry.

In making the experiment a small quantity of mercury is poured into the bend of the tube, and adjusted till the levels in the two limbs stand at the zeros of the scales S and S′.

Then if the section of the short limb of the tube be uniform, the volumes are proportional to the lengths; if the tube be not uniform, then it must be calibrated (see Experiment 17, p. 45).

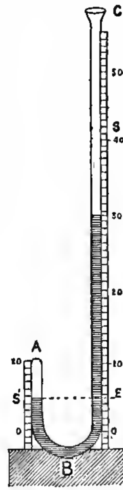


Fig. 37.

The length of the tube occupied by the gas is read off on the scale S′, and the atmospheric pressure noted by the barometer; mercury is now poured in at the end C, and the gas thereby compressed: when the mercury is being poured into the tube it should be tapped with the fingers so as to cause any air bubbles taken down by the mercury to rise to the top. The new length is now read off on the scale S′, and if the mercury stand at the level E in the short limb and at F in the long limb, the new pressure will be the barometric height H

plus the height $EF = h$, and so on for other points. Then if

V = volume proportional to length of gas columns,

$P = H + h$ the total pressure,

$\therefore VP$ = a constant.

Example.—Prove Boyle's law for atmospheric air.

Enter results thus:—

Volume in c.c. or Length in Cms. V.	Pressure in Cms. of Mercury. P.	VP.
14·3	77·4	1106·8
12·3	90·1	1108·2
9·3	119·0	1106·7
7·3	151·2	1103·7

Exercise.

Repeat the above experiment and plot your results in a curve, see Fig. 19, p. 42.

Experiment 43.—**To prove Boyle's law for pressures below that of the atmosphere.**

Instruments required.—A strong iron tube, a glass tube, a centimetre scale, and clean dry mercury.

In Fig. 38, BC is an iron tube or gun barrel, closed at the lower end, having a wide glass funnel or cistern fixed firmly to the top, and supported on a tripod stand, not shown in the figure. The tube is filled with mercury up to near the middle of the glass cistern.

A is a uniform glass tube 100 cms. or so long, closed at A and open at the lower end, which dips in the mercury. S is a scale for measuring the length of the gas column, and also the height of the mercury in the glass tube.

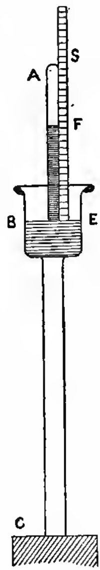


Fig. 38.

In making the experiment the glass tube is entirely filled with mercury, then a sufficient quantity of the air or gas to be tested is introduced; the open end is now closed with the finger, and the tube plunged into the mercury in the iron tube BC.

The glass tube is now pressed down till the level of the mercury in it is the same as that in the cistern, and the length of the gas column in the glass tube measured, which will be proportional to the volume of the enclosed gas at the pressure of the atmosphere then indicated by the barometer.

The glass tube is next raised a little, the enclosed gas dilates, and the mercury in the tube stands at a certain level above that in the cistern. The volume or length of the column of the expanded air and the length of the mercury in the glass tube are again measured. Air, tube, and mercury must all be dry.

The new pressure will be the atmospheric pressure H *minus* the height $EF = h$, and so on for other points.

Then if

$V =$ volume proportional to length of gas column,

$P = H - h =$ total pressure,

$\therefore\ VP =$ a constant.

Example.—Prove Boyle's law for atmospheric air.

Enter results thus :—

Volume in c.c. or Length in Cms. V.	Pressure in Cms. of Mercury. P.	VP.
10·0	69·2	692·0
12·4	56·0	694·4
18·2	38·1	693·4
23·3	29·3	692·7

Exercise.

Repeat the above experiment with hydrogen gas.

Experiment 44.—**Determination of very low pressures of gas by means of the M'Leod gauge.**

Instrument required.—The M'Leod gauge attached to the Sprengel pump.

The M'Leod gauge (Fig. 39) is an arrangement connected with the Sprengel pump, whereby pressures below the indications given by a barometer can be determined. It consists of a globe A, at the upper end of which is a graduated tube B, called the volume tube. A T-piece connects the globe with another graduated glass tube C, called the pressure tube; this tube is in direct communication with the pump and the tube under exhaustion.

Attached to the globe A is a vertical glass tube D, about 80 cms. long, which is connected with the mercury reservoir E by means of a flexible tube. The capacity of the globe A, from a platinum wire *o* to

the lowest division of the volume tube B, having been determined, and the volume of B being known, the ratio of the capacity of B to A is found.

In making the experiment the reservoir E is lowered, so that C is in communication with A and B; after exhaustion, when a measurement has to be made of the pressure of the residual gas, E is raised, the mercury rises in C and A until the whole of the gas in the globe is compressed into B; its pressure is then found by measuring the differences of level of the columns of mercury in the volume and pressure tubes. Dividing this difference by the ratio of the capacities of the globe and volume tubes we obtain approximately the pressure of the gas. If the residual gas is compressed up the volume tube we must of course find the new ratio from the known capacity of the volume tube per division.

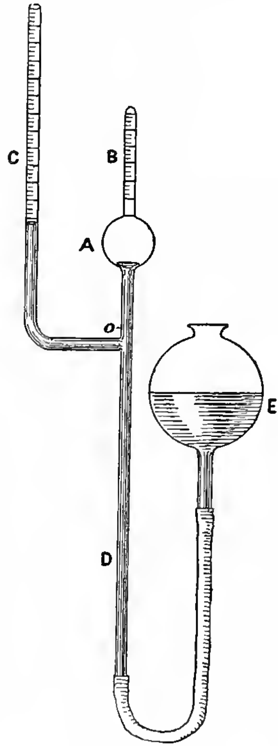


Fig. 39.

On adding the pressure so found to the mercury head (or difference of level in C and B), and once more

dividing this number by the ratio of B to A, the exact pressure of the residual gas is determined.

Example.—After exhaustion the mercury head was found = 5 cms. The ratio of the capacities of B to A was 1 : 42·5, therefore, since

$$VP = V'P', \quad \text{or } P = \frac{V'P'}{V}.$$

i.e.

$$P = \frac{1}{42{\cdot}5} \times 5 = 0{\cdot}117 \text{ cms.},$$

then the corrected pressure $= \dfrac{5{\cdot}117}{42{\cdot}5} = 0{\cdot}12$ cms.

Note.—The M'Leod gauge is subject to error arising from the condensation of the residual gas on the surface of the gauge tube and bulb (see Appendix, § 9).

Exercise.

Examine the construction and working of the Sprengel pump, and make a vacuum tube for experiments with the induction coil or spectroscope as follows: Draw off a piece of glass tube at one end, and then close the other end, fuse a short length of platinum wire into and near each end of the tube, exhaust by the Sprengel pump, and when the mercury falls with a metallic click find the pressure of the residual gas by the M'Leod gauge. Fill a second similar tube with pure hydrogen, exhaust, and determine the pressure in the same way.

CHAPTER VI

MEASUREMENT OF FORCE

FORCE is the mutual action of two bodies upon one another, and its definition is implicitly contained in Newton's First Law of Motion—viz., force is any cause which alters, or tends to alter, a body's state of rest or of uniform motion in a straight line. The sense of muscular exertion gives us our primitive idea of force, and whatever else is capable of producing a similar effect is said to exert force.* The Second Law of Motion expresses the action of force on matter, and teaches us how to measure force by observation of its effects. The Third Law of Motion shows that wherever force exists in nature it is invariably accompanied by an equal and opposite force, the two forces constituting a *stress.* A body in motion encountering no resistance is not exerting force. Force is in every case a transference of, or a tendency to transfer, energy from one body to another.

The effects of force on matter are twofold, either (*a*) producing change of motion, that is, *acceleration*,

* "The least ambiguous meaning of force is simple pressure or tension."

or (*b*) producing change of size or shape, that is *strain.* Either the amount of acceleration or of strain produced affords a measure of force. In the following experiments forces will first be measured by the strain they are able to produce, as in a spring balance, and then the force of gravity will be measured by the acceleration "g" it can produce * (see Table IV.).

Experiment 45.—**Measurement of force by means of a dynamometer.**

Instruments required.—A spring balance and other forms of dynamometer.

Force can be measured indirectly by means of a spring balance, if the value of the acceleration due to gravity at the place be known. Thus if $g = 980$ cms. per second per second, then 40 grammes weight $= 40 \times 980 = 39200$ dynes (force); or if "g" $= 32$ feet per second per second, then a weight of 3 pounds $= 3 \times 32 = 96$ poundals (force) (see pp. 9 and 10).

Exercises.

1. Take a spring balance or other form of dynamometer graduated in grammes or kilogrammes, and make a series of observations by putting on pound weights, tabulate the results, and reduce them to dynes.

2. Take a piece of thin steel wire and make it into a spiral spring by winding it on a small mandril; fix one

* For a fuller discussion of the subject of force, see Professor O. Lodge's *Elementary Mechanics*, or Principal Garnett's admirable text-book on *Elementary Dynamics*.

end of this spring to a firm support, and hang a scale pan with pointer on the other end. Now add gramme weights to the scale pan, noting the extension of the spring for every additional gramme, and plot the results on millimetre paper.

Experiment 46.—**Measurement of centrifugal force, etc., with a whirling table.**

Instruments required.—A whirling table with accessories.

Owing to inertia, as defined in the First Law of Motion, a moving body continues in a state of uniform motion in a straight line; hence to keep a body moving in a curve a force must act upon the body, which can be shown to be directed towards the centre of curvature, if the speed be uniform; this is the so-called centripetal force, as seen in the tension of a string that holds a whirling body. But by the Third Law of Motion an equal force tending outwards must be exerted on the body to which the other end of the string is attached; this is the so-called centrifugal force, the two forces being opposite aspects of the stress on the string. If

F = the centrifugal or centripetal stress,

v = the velocity of rotation of the body,

m = the mass of the body,

r = its distance from the axis of rotation,

then
$$F = \frac{mv^2}{r}.$$

Exercises.

1. With the whirling table revolving uniformly, find by means of a registering dynamometer the centrifugal force exerted by a body of known weight placed at different distances from the centre of the table.

2. In the same way determine the centrifugal force exerted by bodies of different weights when placed at the same distance from the centre of the table.

3. Fix a strong wide glass tube half full of liquid on the centre of the table, and note the form of the surface of revolution of the liquid when the table is revolving uniformly. Repeat with water, oil, and mercury.

4. Fix a glass globe full of water on the centre of the table. Float a wax ball on the water; when the table is revolving uniformly, note that the ball can be sunk to any depth and will remain there in equilibrium.

Experiment 47.—**To determine the brake horse-power by a friction dynamometer.**

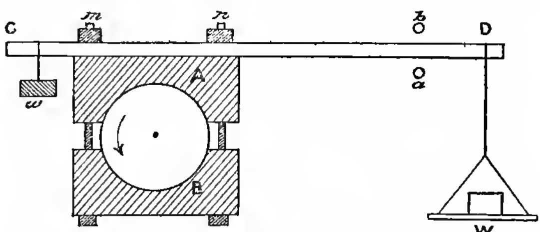


Fig. 40.

Instruments required.—A Prony brake, a stop-watch, and a speed-counter.

The Prony brake is a form of absorption dynamometer,

and consists (Fig. 40) of two pieces of wood A B hollowed out to fit a pulley or wheel on the engine or machine to be tested. These can be clamped to any desired extent by means of the two screw nuts m n, a stout rod CD is attached to the upper piece of wood having a scale pan at one end D.

Before making the experiment the scale pan must be counterpoised by a weight w; a known weight W is now put on the scale pan, the engine started and brought to the desired speed, the screw nuts m n being at the same time tightened or the weight W altered until it is just kept balanced, that is when the rod CD is between the stops a b. The number of revolutions per minute is at the same time taken with the speed-counter and stop-watch. If

n = revolutions per minute,

l = distance in feet from the centre of the pulley to where the weight W acts on the lever,

W = weight in pounds in the scale pan, then

$$\text{H-P.} = \frac{2\pi n l W}{33000}.$$

Example.—Find the horse-power of the laboratory water motor when driving a lathe.

$$n = 162, \quad l = 2 \text{ ft.}, \quad W = 4 \text{ lbs.}$$

$$\therefore \text{ H-P.} = \frac{2\pi \times 162 \times 2 \times 4}{33000} = \tfrac{1}{4} \text{ horse-power nearly.}$$

A simpler mode of determining the horse-power is to take a stout cord or rope and lap it one or two times round the fly-wheel or pulley of the engine to be tested,

as in Fig 41. A Salter's balance S is fixed to the floor and to one end of the rope; a small weight W being

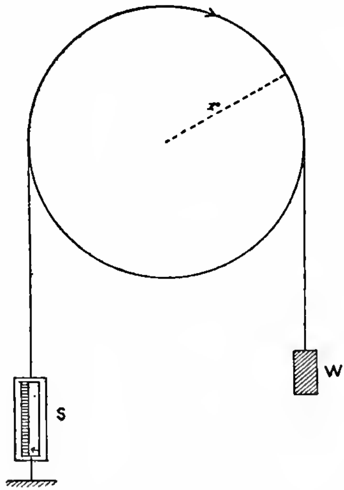


Fig. 41.

hung at the other end of the rope. The wheel revolves as shown by the arrow, so as to stretch S, and the speed taken as before. If

n = revolutions per minute,

R = radius of the fly-wheel,

W = reading on the Salter's balance,

W′ = weight on the other end of rope, then

$$\text{H-P.} = \frac{2\pi n \text{R}(\text{W} - \text{W}')}{33000}.$$

Note.—If N = revolutions per *second*, R = radius of fly-wheel in *cms.*, W and W′ = weights in *grammes*, then

$$\text{H-P.} = \frac{2\pi \text{RN}(\text{W} - \text{W}')}{7{\cdot}6 \times 10^6}.$$

Exercises.

Find the horse-power under varying loads of—

1. An electric motor.
2. The Laboratory gas-engine.

Experiment 48.—**To determine the value of 'g' by Atwood's machine.**

Instruments required.—Atwood's machine, a stop-watch, and a centimetre scale.

Atwood's machine (Fig. 42) consists of a frictionless pulley A, with a fine string passing over it, and having two equal weights M attached to it. Another small weight m, called the rider, is put on one of the large weights, and thus sets the system in motion. When the weight $M+m$ has fallen a certain known distance, a ring R takes the rider m off and allows the large weight M to travel onwards through another known distance to the platform P; the time of travelling this latter distance being carefully noted.

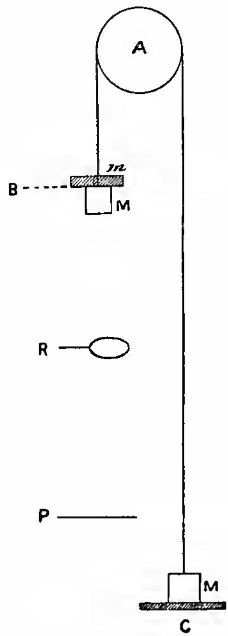


Fig. 42.

In making the experiment first find by trial the smallest weight, w, that will set the wheel-work in motion, this gives the correction for the friction of the wheel-work.

Now add the rider m, and note the time taken for it to fall from R to P; repeat with a heavier rider m', noting the time again. This double experiment enables the momentum of the wheel-work to be taken into account.

A seconds pendulum is usually attached to the machine; the time may be taken by its means, or by an ordinary watch—Experiment 21 (2); or more accurately by a stop-watch reading to fifths of a second, or by a chronograph. Now measure BR and RP.

In order to start the motion, one weight should rest on the platform C, and, when all is steady, the hand removed from the string AC. If

g = acceleration due to gravity,

s = distance fallen *before* m is removed,

s_1 = distance fallen *after* m is removed,

t_1 and t_2 = times taken to fall from R to P, after m and m' are removed respectively,

w = weight required to overcome the friction,

then
$$g = \frac{(m - m')s_1^2}{2s\{mt_1^2 - m't_2^2 + w(t_2^2 - t_1^2)\}}.$$

(See Appendix, § 10.)

Example.—Find the value of "g" by Atwood's machine.

$s = 50$ cms. $m = 31{\cdot}2$ grammes.

$s_1 = 110$ cms. $m' = 15{\cdot}6$ grammes.

$t_1 = 1{\cdot}7$ secs. $w = 0{\cdot}57$ gramme.

$t_2 = 2{\cdot}4$ secs.

Putting these values in the above equation and reducing we get

$$g = \frac{15{\cdot}6 \times 110^2}{100 \times 1{\cdot}956} = 965 \text{ cms. per sec. per sec.}$$

As "g" in Dublin is 981·3, this is at least 1·6 per cent too low. It will therefore be seen that this method of determining g is far less accurate than the pendulum experiments which follow; this is mainly on account of the experimental difficulty in obtaining the exact value of w, the friction of the wheel-work. Thus, making w 0·58 gramme instead of 0·57 brings the result as much as 4·2 per cent too low.

Exercise.

Determine the value of "g."

Experiment 49.—**Determination of the laws of falling bodies.**

Instruments required.—The same as in the last experiment, or Morin's apparatus.

The preceding experiment shows that as an instrument of precision the ordinary form of Atwood's machine is of small value. For the purpose of investigating the laws of falling bodies it is, however, of great service to the student, as by its means the force of gravity is diluted, so that its effects can be observed within a limited fall.

The relationship between the space traversed by a falling body and the time taken to traverse it can also be verified by means of Morin's apparatus. This consists of

a vertical drum or cylinder covered with paper, and made to rotate on its vertical axis with a uniform velocity. The falling weight, kept parallel to the cylinder by guides, has a pencil attached to it which marks the drum as it falls. A curve is thus drawn, due to the composition of the uniform horizontal motion of the cylinder with the uniformly accelerated vertical motion due to gravity. On removing the paper from the cylinder this curve is found to be a parabola OP (Fig. 43). With one or other of these instruments the following relations may be proved :—

$$\text{Space } (s) = \tfrac{1}{2}gt^2 = \tfrac{1}{2}vt,$$

$$\text{Velocity } (v) = gt = \sqrt{2gs} = \frac{2s}{t},$$

$$\text{Time } (t) = \sqrt{\frac{2s}{g}} = \frac{v}{g} = \frac{2s}{v}.$$

Example 1.—Show by Atwood's machine that $s = \frac{1}{2}gt^2$, taking $g = 32{\cdot}2$ in British units.

The mass on one side of the pulley was M and on the other side $M + m = M'$, hence the total mass to be moved was $M' + M$, and the moving force $M' - M$. The acceleration $a = \dfrac{\text{force}}{\text{mass}} = \dfrac{M' - M}{M' + M} = \dfrac{23 - 22}{23 + 22} = \frac{1}{45}$ of g.* The time taken to fall from rest through the space, s, was found to be 3 seconds. As $a = \frac{1}{45}g$, therefore $s = \frac{1}{2} \times \frac{1}{45} \times 32{\cdot}2 \times 3^2 = 3{\cdot}22$ feet. By direct measurement the space was 38 inches.

* To the *denominator* of the fraction must be added the small weight necessary to overcome the friction of the wheel-work, and the value of which must be found by trial before making the experiment.

Example 2.—Prove by Morin's machine that the space described by a falling body is proportional to the square of the time taken to fall.

The figure obtained on the paper was the curve OP, shown in Fig. 43. Draw the vertical OY and the horizontal line OX. Divide OX into equal parts, as 1, 2, 3, etc.; from these points draw vertical lines cutting the curve at the points *a*, *b*, *c*, etc.; from OY draw per-

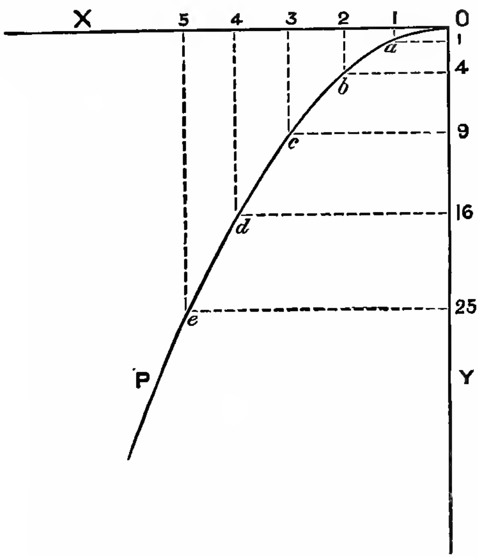


Fig. 43.

pendiculars meeting the curve at these points. As OX represents the time taken for the body to fall through the space OY, the successive spaces from O along OY will be found proportional to the squares of the distances

from O along OX. Hence, as g is constant, s is proportional to t^2.

Exercises.

1. By Atwood's machine prove that $v = 2s/t$.

2. Show that the velocity is being accelerated before and remains uniform after the rider is removed from the falling weight.

3. Show by Atwood's machine that the velocity generated by a constant force is proportional to the time during which the force has acted.

4. Show also that, the mass remaining the same, the velocity generated in unit time varies directly as the force, and with the force constant the velocity varies inversely as the mass.

5. With Morin's apparatus prove that the distance fallen from rest, divided by the square of the time taken to fall, is a constant.

Note.—With a *freely falling body*, and a chronograph, the value of g may also be determined, and the laws of falling bodies deduced (see Experiment 21). For this purpose simple electro-magnetic appliances attached to Willis's apparatus * give excellent results.

Experiment 50.—**To prove the isochronism of the vibrations of a torsional pendulum.**

Instruments required.—A wire clamped above, and a weight with pointer moving on a graduated circle below.

By Hooke's law, within the limits of elasticity the "torque" of restitution is proportional to the amount of

* See Sir Robert Ball's *Experimental Mechanics.*

distortion. This causes the oscillations which are set up on the removal of constraint to be performed in equal times, that is to say they are isochronous.

Exercises.

1. Prove the isochronous character of the oscillations of a torsional pendulum.

Twist the wire through an angle of 45°, and, by the method of passages (Experiment 19), find the time of a single oscillation, noting the angle at the close of the experiment; now twist the wire through 90°, and again determine the time of a single oscillation, noting the angle at the close.

2. Prove by a similar experiment the isochronism of an ordinary simple pendulum vibrating through a small arc.

Experiment 51.—**To prove experimentally the laws of the simple pendulum; determination by this means of the value of 'g.'**

Instruments required.—A simple pendulum, a centimetre rule, and a stop-watch.

A simple pendulum is theoretically a massive point suspended from a rigid support by a massless thread, and swinging through a small arc.

This condition is approached in practical work by suspending a lead bullet, say 1 cm. in diameter, by a fine silk thread, the centre of gravity of the bullet being taken as the centre of oscillation.

In making the experiment, the length of the pendulum (*i.e.*, the distance from the point of support to the centre of the bullet) is varied, and the corresponding period of vibration observed.

The pendulum is made to vibrate through an arc not exceeding 3° on each side of the vertical—*i.e.*, a horizontal length of about 5 cm. on each side in a pendulum 1 metre long.

A great number of observations, or sets of observations, are taken, and the mean time of vibration determined. Then if

T = period of vibration in seconds,*

l = length of the pendulum in cms.,

g = acceleration due to gravity,

$\pi = 3{\cdot}1416$,

$$T = 2\pi\sqrt{\frac{l}{g}},$$

$$T^2 = 4\pi^2\frac{l}{g},$$

$$\therefore \frac{l}{T^2} = \frac{g}{4\pi^2} \text{ (a constant).}$$

(See Appendix, § 11.)

Example 1.—To prove experimentally the pendulum laws.

* We use the term *vibration* to indicate a double oscillation, that is a complete movement to and fro, a "swing-swang," as it has been called.

Enter results thus:—

Number of Experiment.	T Secs.	T^2	l Cms.	$\frac{l}{T^2}$
1	1·84	3·38	83·4	24·67
2	1·76	3·11	76·7	24·66
3	1·65	2·73	67·6	24·76
4	1·59	2·54	62·7	24·68
5	1·13	1·27	31·5	24·69

Example 2.—To find the value of "g" by the simple pendulum, we have

$$g = \frac{4\pi^2 l}{T^2}.$$

The time and length being observed very accurately, we obtained

$l = 230$ cms.,

$T = 3{\cdot}04$ secs., which was the mean of 10 sets of 10 vibrations each,

$$\therefore g = \frac{4 \times 9{\cdot}87 \times 230}{3{\cdot}04^2} = 982{\cdot}5.$$

The value of "g" in Dublin is 981·32 cms., hence the result in the foregoing experiment is 0·12 per cent too large (p. 38); thus $982{\cdot}5 - 981{\cdot}32 = 1{\cdot}18$, and

$$981{\cdot}32 : 100 = 1{\cdot}18 : 0{\cdot}12.$$

For an accurate determination the result thus obtained needs to be corrected—

(i.) Should the mean amplitude of the swing θ exceeds 3°.

(ii.) In order that the length may be reduced to that of the equivalent simple pendulum; thus if the bob be a sphere the corrected length is $l+\frac{2r^2}{5l}$ (see Appendix, § 12).

Hence the formula for "g" becomes

$$g=\frac{4\pi^2}{T^2}\left(l+\frac{2r^2}{5l}\right)\left(1+\frac{1}{4}\sin^2\frac{\theta}{2}\right)^2.$$

Other corrections are—

(iii.) Buoyancy of the air on the bob.
(iv.) Resistance of the air.
(v.) Dragging of the air by the bob.
(vi.) Want of rigidity of the support.

Exercises.

1. Prove by experiment the laws of the simple pendulum, taking l as 20, 40, 80, and 100 cms. respectively.

2. Find the value of "g" by the pendulum, applying corrections (i.) and (ii.)

Experiment 52.—**Determination of the value of 'g' by Kater's pendulum.**

Instruments required.—A Kater pendulum, a millimetre scale, a clock or stop-watch.

A simple form of Kater's compound pendulum (Fig. 44) consists of a bar of seasoned hardwood AB, with

two weights attached to it, M and *m*. The larger weight M is fixed permanently to the bar, the other weight *m* can be moved up or down the bar and clamped at any desired position. C and D are two steel knife-edges fixed in the bar, and the pendulum can be made to vibrate about either one of them, the knife-edges resting on agate plates.

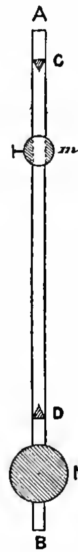


Fig. 44.

In making the experiment the pendulum is caused to vibrate about the knife-edge C, and the vibrations accurately observed. It is then made to vibrate about the knife-edge D, and the vibrations again carefully observed.

If the number of vibrations in equal times are not the same about the two knife-edges, the small weight *m* is moved slightly and the observations again made; the weight *m* being always moved *towards* that knife-edge about which the time of vibration is the greater.

Having by adjustment got the times of vibration about the two axes equal, measure very accurately the distance between the knife-edges, which will be the length of the equivalent simple pendulum.

(A simple form of Kater's pendulum, which with care gives fairly good results, can be made with a thick, flat, piece of heavy wood, about 1 metre long and 5 cms. broad. A hole is made near each end and uniform pieces of glass rod inserted into the holes, which serve as knife-edges, and in place of the agate plates, two pieces of the same glass rod are fixed firmly on a suitable support.

The adjustment to equal times of vibration can be done by slipping small strips of lead up or down the rod.) If

l = distance between the knife-edges,

T = period of vibration,

g = acceleration due to gravity,

$$\text{then } g = \frac{4\pi^2 l}{T^2}.$$

Example.—To find the value of "g" by Kater's pendulum.

By trial and error the times of vibration about the two axes were found to be 1·74 sec., and the corresponding distance between the knife-edges 75·3 cms.

$$\therefore\ g = \frac{4\pi^2 \times 75{\cdot}3}{1{\cdot}74^2} = 981{\cdot}9.$$

If the times of vibration about the two centres are not perfectly the same, it is convenient in an experiment to use the following formula—

Let h and h_1 be the distances of the centre of gravity of the pendulum from the two knife-edges, t and t_1 the corresponding times of vibration, then

$$g = \frac{4\pi^2(h^2 - h_1{}^2)}{(ht^2 - h_1 t_1{}^2)}. \text{ *}$$

Exercise.

Find the value of "g" by means of a Kater's pendulum (1) when the periods are in perfect agreement; (2) when the periods are slightly different.

* For the proof of this equation, see Routh's *Rigid Dynamics*, Part I, p. 80.

Experiment 53.—**To illustrate the composition of two simple harmonic vibrations in rectangular directions by means of Blackburn's compound pendulum.**

Instruments required.—A Blackburn pendulum and a centimetre scale.

Blackburn's pendulum (Fig. 45) consists of a combination of two pendulums on the same string suspended from the one support M.

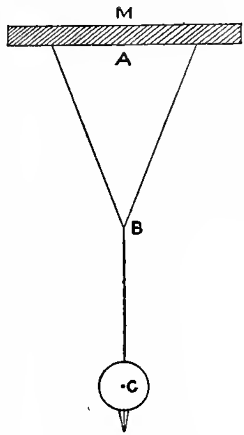


Fig. 45.

The pendulum of length AC vibrates in the plane perpendicular to the plane of the paper, and the other of length BC vibrates in the plane of the paper.

If BC is vibrated alone, it does so from B as a fixed point.

Now if the bob C be drawn aside in the direction of the bisector of these two planes, it will take the path due to the compounding of the two separate motions. In making the experiment the bob C is usually a disc of lead with a fine nosed funnel inserted into it, through which a stream of sand flows and thus traces out the path of the bob.*

* These curves, together with the mode of drawing them geometrically, will be given in the part on Sound, and for the theory of the Blackburn pendulum see Everett's *Vibrating Motion and Sound*, chap. vi.

From the simple pendulum law we have

$$T = 2\pi\sqrt{\frac{l}{g}},$$

$$\therefore\ l_1 : l = T_1^2 : T^2,$$

$$l_1 = \frac{lT_1^2}{T^2}.$$

Example.—Find the lengths of a compound pendulum when the periods of the component pendulums are in the ratio 5 : 6.

$$\text{The length } l = AC = 100 \text{ cms.},$$

$$\therefore\ l_1 = BC = \frac{100 \times 5^2}{6^2} = 69{\cdot}44 \text{ cms.}$$

Exercise.

Find the lengths and obtain the tracings of the paths of the bob when the periods are in the ratio of (i.) 2 : 3, (ii.) 3 : 4, (iii.) 4 : 5.

Experiment 54.—**To find the centre of percussion of a body.**

Instruments required.—A rough plank of wood, thread, and a bullet.

The centres of oscillation and suspension in a compound pendulum are reciprocal. The centre of oscillation is identical with another remarkable point called the centre of *percussion*, which may be defined as follows: If a body be suspended from any point in it, the centre of percussion is such a point in the body that when an

impulse is received whose line of action passes through this point in a direction perpendicular to the straight line joining this point to the centre of suspension, no reaction or impulse is caused at the centre of suspension.

This point is of much practical importance, for a blow delivered through the centre of percussion produces simply rotation about the centre of suspension of the body without instantaneous strain on the support. Hence, for example, a door-stop should be placed at the centre of percussion of the door, otherwise the hinges will be strained.

In making the experiment, suspend the body by a pin so as to oscillate about the axis of suspension, and from the same support hang a simple pendulum, and adjust its length until its period of vibration synchronises with that of the body. Then the length of this equivalent simple pendulum is the same as the distance between the centres of suspension and oscillation of the body, and the latter point gives us the centre of percussion.

Example.—Find the centre of percussion of a piece of wood 120 cms. long, 3 cms. broad, and 2 cms. thick, when suspended by a pin at a distance of 2 cms. from one end.

By trial and error the length of the simple pendulum which synchronised with the rod was found to be 78·5 cms., therefore the centre of percussion is 78·5 cms. from the point of suspension of the rod, that is about two-thirds its length from the support.

Exercises.

1. Find the centre of percussion in the Ballistic pendulum used in the next experiment.

2. Find the centre of percussion of an irregular shaped plank.

Experiment 55.—**To determine the velocity of a bullet by means of the Ballistic pendulum.**

Instruments required.—Ballistic pendulum, a gun, stop-watch, and centimetre rule.

The Ballistic pendulum (Fig. 46) consists of a heavy bob of wood C, supported by a light frame-work from a horizontal knife-edge A. D is a point on the bob in the line of the axis to which a tape is attached for measuring the chord of the arc of oscillation of the pendulum when it has been disturbed by the impact of the bullet.

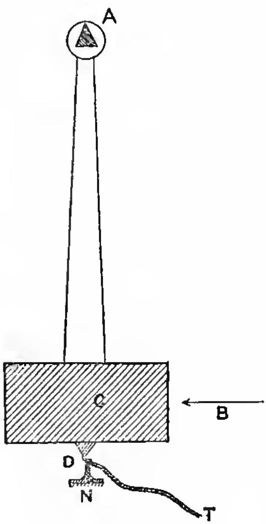


Fig. 46.

A clip N is fixed on the table or stand, quite close to D, when the pendulum is at rest, and serves to put a small amount of friction on the tape when it is being drawn out by the swing of the pendulum; on the return of the pendulum the length of tape drawn out remains unaltered.

In making the experiment the bullet is fired in the direction B, perpendicular to the line through the knife-edge and centre of percussion of the pendulum. If

M = mass of the pendulum bob with the bullet in it,
m = mass of the bullet,
h = distance from the centre of percussion of the bob to the knife-edge,
h_1 = distance of the line of fire from the knife-edge,
L = distance of attachment of tape from the knife-edge,
S = length of tape drawn out,
T = half period of equivalent simple pendulum,
g = acceleration due to gravity,
$\pi = 3{\cdot}1416$,
V = velocity of the bullet,

then $$V = \frac{MhgTS}{mh_1L\pi} \quad \text{(Appendix, § 13).}$$

Example.—Find by the Ballistic pendulum the velocity of a bullet shot from an air-gun.

By experiment we have

$$M = 6000 \text{ grams.,} \qquad m = 6 \text{ grams.}$$
$$h = h_1 = 59{\cdot}5 \text{ cms.,} \qquad L = 71 \text{ cms.}$$
$$S = 5{\cdot}2 \text{ cms.,} \qquad T = 0{\cdot}75 \text{ secs.}$$

$$\therefore V = \frac{6000 \times 59{\cdot}5 \times 980 \times 0{\cdot}75 \times 5{\cdot}2}{6 \times 59{\cdot}5 \times 71 \times 3{\cdot}1416}$$

$$= 17140 \text{ cms. per second.}$$

Exercises.

1. Vary the air charge in the gun and repeat the above experiment.

2. Hang a heavy ball up by a thread and let it fall against the pendulum, then calculate the velocity with which it strikes the bob.

Experiment 56.—**To determine the moment of inertia of a body about a given axis.**

Instruments required.—A body of known moment of inertia, a stop-watch, and a centimetre scale.

Hang up the body whose moment of inertia is required by a wire firmly fixed to it and passing through the axis of rotation and centre of gravity of the body, and take the period of vibration.

Then add the body of known moment of inertia, so that this compound body will vibrate round the same axis, and again note the period of vibration.

Now if

K = moment of inertia required,
k = moment of inertia of known body,
t_1 = period of unknown body,
t_2 = period of compound body,
m = couple exerted by the wire when twisted one radian (57·3°) from rest,

then $$t_1 = 2\pi\sqrt{\frac{K}{m}} \quad \text{(in the first case)},$$

$$t_2 = 2\pi\sqrt{\frac{K+k}{m}} \quad \text{(in the second case)},$$

$$\therefore\ K = k\frac{t_1^2}{t_2^2 - t_1^2}.$$

Example.—Find the moment of inertia of an irregular shaped weight, using a lead disc, 8 cms. diameter, and of weight $M_1 = 3176$ grammes, as the body of known moment of inertia (see Table XIV. and Appendix, § 14).

$$k = \tfrac{1}{2}MR^2 = \tfrac{1}{2} \times 3176 \times 4^2 = 25408,$$

$$\therefore\ K = 25408 \frac{14{\cdot}4^2}{15{\cdot}7^2 - 14{\cdot}4^2} = 134662.$$

Exercises.

Find by experiment the moment of inertia (1) of a rectangular rod; (2) of a small fly-wheel rotating round the axis of figure, and compare the results with those got by calculation.

Note.—In the foregoing experiment, and also in Experiment 64, the moment of inertia of the vibrating body must be altered without changing the force tending to bring the displaced vibrator back to its original position. This can be done either (1) by adding suitable masses whose moments of inertia can be calculated, such as discs, slotted to go past the suspending wire, or (2) by changing the configuration of the vibrator with reference to the axis of rotation, as in Maxwell's needle, Experiment 66.

CHAPTER VII

MECHANICAL PROPERTIES OF MATTER: SOLIDS

Elasticity—Tenacity—Resilience

Elasticity is that property of a body in virtue of which it recovers from a deformation.

For every solid body there is a limit beyond which, if it be deformed, it will not entirely recover itself; this is called the *limit of elasticity*, within this limit it will return to its original size and shape, that is the strain will disappear on the removal of the stress.

Strain is the change in the size or shape of a body.

Stress is that which produces a strain.

The ratio, *stress* divided by *strain*, is the general expression for the coefficient of elasticity of a body.

For *isotropic* bodies, *i.e.* those which have similar properties in all directions, there are two coefficients.

(i.) The coefficient of the *clasticity of volumc*, which is the only one possessed by fluids.

(ii.) *Simple rigidity*, which is confined to solid bodies, or those which resist change of shape and change of volume.

Shear, or *shearing strain*, is change of form without change of volume.

Shearing stress is that which produces shearing strain.

$$\textit{Coefficient of simple rigidity} = \frac{\text{shearing stress}}{\text{shearing strain}}.$$

Young's modulus of elasticity is stress per unit area divided by strain per unit length. This is usually determined by stretching or flexure.

Modulus of torsion of a wire is the *couple* required to twist one end of the wire of unit length through unit angle, the other end being fixed.

Tenacity is the breaking force per unit area acting by direct tension.

Experiment 57.—**To determine the tenacity of a wire, together with its elastic limit.**

Instruments required.—The same as in the next experiment or a testing machine with dynamometer.

The tenacity of a body is the greatest longitudinal stress per unit of cross-sectional area which it can bear without breaking; bodies which have the highest coefficient of elasticity are not necessarily the most tenacious. The elastic limit of a body has been defined above; in engineering it is essential that this limit should not be exceeded nor even too nearly approached. The ductility of a body is the amount of permanent change of form under stress that it can undergo without rupture, and for wires of the same cross-section may be relatively measured by the percentage elongation or extension per unit length of the body just before it breaks.

In making the experiment the wire under test is firmly held in the jaws of a strong clamp attached to the wall, or gripped in the jaws of the testing machine; the smallest stress that will straighten the wire having been put on, the length and diameter of the wire are carefully measured. Tension is now gradually applied and the corresponding increments in length are noted until the elastic limit is reached; when this is passed it will be noticed that the wire is permanently elongated; the index hand of the dynamometer now remains stationary during the application of additional stress and the wire begins to draw, finally it breaks, the length of the wire being noted just before or after rupture.* Thin wires have a higher tenacity per unit area than thick wires of the same material. If

$T =$ tenacity in grammes per square cm.,
$P =$ breaking weight in grammes,
$l =$ original length of the wire in cms.,
$l' =$ final " " " "
$A =$ cross-section area of the wire in square cms.,

then $$\frac{l'-l}{l} = \text{extension per unit length } \epsilon,$$

and $$T = \frac{P}{A}.$$

Example.—Find the elastic limit, the tensile strength, and the extension per unit length of a thin annealed copper wire; radius $= 0{\cdot}038$ cms.

* Bottomley has noticed that when the stress is added slowly, with *intervals of rest between*, the tenacity of iron wire is greater than when no rest is allowed. The lifting power of a magnet behaves similarly.

Enter results thus—

$P = 11500$ grammes, $\qquad l = 101$ cms.,

$A = \pi r^2 = 0{\cdot}0045$ sq. cms., $\qquad l' = 127$ „

$$\therefore T = \frac{11500}{0{\cdot}0045} = 2{\cdot}54 \times 10^6 \text{ grammes per square cm.}$$

$$\epsilon = \frac{127 - 101}{101} = 0{\cdot}257 \text{ for this wire.}$$

It was found that with a weight of 3500 grammes the elastic limit was reached when a permanent elongation of 0·2 cm. occurred (see Table XII.).

Note.—To convert grammes per square cm. to pounds per square inch, divide by 70·31.

Exercises.

1. Find the elastic limit, the tenacity, and the percentage elongation of specimens of German silver and iron wire.

2. Make similar experiments with wires of the same metal annealed and hard.*

3. The tenacity of unspun silk is stated to be 500,000 grammes per square mm., nearly ten times that of steel. Find the tenacity of unspun silk, and of silk, cotton, and linen thread, also of a quartz fibre and of bass matting.

Experiment 58.—**Determination of the brittleness of a given wire.**

Instruments required.—A scale pan, gramme weights, a metre scale (reading to millimetres) with a vernier

* Metals are annealed by heating to redness and cooling quickly, with the exception of steel, which is hardened by heating to whiteness and plunging in cold water.

sliding the whole length of the scale, or a pointer moving over the scale, and a fixed rigid clamp for the wire.

Fix a wire, say two metres long, firmly at one end, let it hang vertically with a scale pan at the other end; put weights in the pan, and note the elongation of the wire for each weight, the weights being gradually increased till the wire breaks.

If M be the weight, including that of the pan, which stretches the wire just beyond the elastic limit (that is the smallest weight which produces a permanent set in the wire), and N the breaking weight of the wire; then the ratio M/N is called the brittleness.

If this ratio M/N be unity, we may call the body perfectly brittle, and the brittleness of any other substance will be expressed as a fraction or a percentage.

In making the experiment, a known weight is put in the scale pan, and the elongation of the wire noted by means of the scale and vernier or pointer; the weight is now taken off, and the position of the pointer again observed to see if it returns to its original zero.

This process is repeated with gradually increasing weights until the elastic limit is just passed, that is to say, when upon removal of the weights, the pointer does not return to zero. Weights are now gradually added to the scale pan, and the elongation for each additional weight carefully noted till the wire breaks.

The results are then plotted in a curve, with the *stresses* as *abscissæ* and the *strains* as *ordinates*, as in the example on the next page.

Example.—Determine the brittleness of a specimen of German silver wire 0·375 mm. diameter.

Enter results thus:—

Stress (Kilogrammes).	Strain (Centimetres).
1·0	0·25
2·0	0·50
3·0	0·75
3·5	0·85 elastic limit
4·0	1·60
4·5	3·30
5·0	6·50
5·5	10·50
6·0	15·70
6·5	23·00
6·6	Wire broke

The above results are plotted in the curve (Fig. 47).

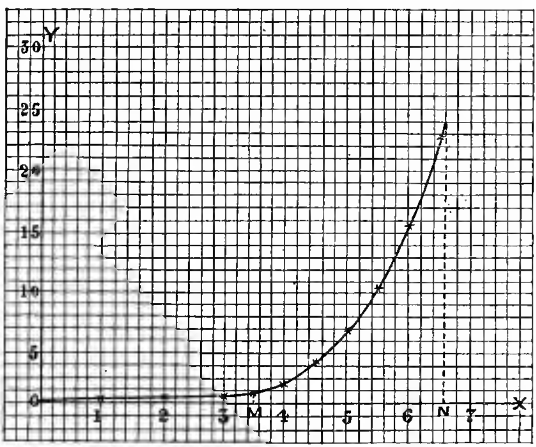


Fig. 47.

The brittleness $= \frac{OM}{ON} = \frac{3 \cdot 5}{6 \cdot 6} = 0 \cdot 53$, or 53, perfect brittleness being 100. The reciprocal of this ratio may be taken as a measure of *the ductility* of the wire.

Exercises.

1. Find the brittleness of specimens of steel, iron, and copper wire.
2. Ascertain whether wires of the same metal, and in the same condition, have the same brittleness and tenacity when their sectional areas alone are different.

Experiment 59.—**To determine Young's modulus by the stretching of a wire.**

Instruments required.—A clamp, scale pan, weights, two verniers, and a cathetometer.

The wire under experiment is clamped to a rigid support,* and a mark made near the lower end of the wire; this can be accomplished either by a fine wire, a fine line on paper fixed to the wire, or a scratch on a metal plate rigidly attached to the wire, and the elongation read off by means of the cathetometer.

A simpler method, which does not require the use of the cathetometer, and gives quick readings, is to have instead of a mark a small vernier attached to the wire, moving alongside of its corresponding scale, which latter must be independently supported (Fig. 48). If both are

* A thread—on which is a mark—with a small weight attached, if hung from the same support as the wire, will indicate any want of rigidity in the support by the motion of the mark when the wire is loaded.

hung from the same support, as shown in the figure, the bending of the support can be neglected.

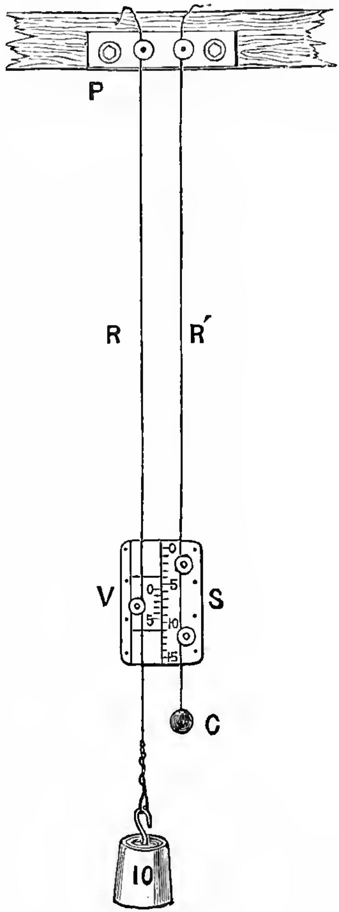


Fig. 48.

A scale pan is hooked on to the lower end of the wire, and just sufficient weight put in the pan to keep the wire straight; then the stretching weight (which should be well within the limits of elasticity of the wire) is put on the pan, and the elongation noted. This weight is then taken off and put on again, once or twice, to make sure that no permanent elongation of the wire has taken place—the readings *on* and *off* being taken each time. The length and diameter of the wire are then measured.

Another method, which also excludes any want of rigidity in the support to which the wire is clamped, is to employ two verniers, one near the top of the wire and the other near the bottom; the difference in the readings of the verniers

gives the total elongation between the two points where the wire is clamped to the verniers. If

E = the Young's modulus,

F = stretching force in dynes, *i.e.* the stretching weight in grammes multiplied by "g,"

L = length of the wire in cms.,

e = elongation of L produced by F,

A = cross-section in sq. cms.,

then $$E = \frac{FL}{Ae} \text{ (see p. 135).}$$

Or Young's modulus is the amount of force that would double the original length of our wire, if its sectional area were 1 square cm., provided it were possible to do this without breaking the wire.

Example.—Find the Young's modulus for stretching of an iron wire, using the two verniers.

By measurement, $L = 355{\cdot}3$ cms.,

$$F = 5000 \times 981 \text{ dynes.}$$

$$A = \pi r^2 = (\pi \times {\cdot}0375^2) \text{ sq. cms.,}$$

The upper vernier reads 0·15 mm.,

The lower vernier reads 3·25 mm.;

$$\therefore e = 3{\cdot}1 \text{ mm.} = 0{\cdot}31 \text{ cms.,}$$

$$E = \frac{5000 \times 981 \times 355{\cdot}3}{\pi \times {\cdot}0375^2 \times 0{\cdot}31}$$

$$= 1{\cdot}27 \times 10^{12} \text{ dynes per square cm.}$$

Note.—If instead of dynes per square cm. the result be required in grammes per square cm., omit the value of "g" in the formula.

Exercises.

1. Find the Young's modulus of a steel pianoforte wire (see Table XI.).

2. Make a stress-strain diagram, like Fig. 47, by gradually loading a soft iron wire (say No. 20 B.W.G.) until it breaks, observing tension, elongation, and elastic limit.

3. Do the same for a piece of india-rubber, noting diminution of the cross-section of the rubber (see Experiment 61): reduce the diagram to correct for this.

Experiment 60.—**To determine Young's modulus by the flexure of a rod.**

Instruments required.—Two rigid supports, a scale pan, weights, and a cathetometer.

The rod under experiment is laid on two firm supports, and a scale pan hung from the middle of the rod, having a mark by which the flexure is measured with the cathetometer.

Instead of the rod being supported at both ends it may be firmly fixed at one end, the other end being free, and the bending force applied at the free end, the flexure being determined as above with the cathetometer. If

E = Young's modulus by flexure,

L = length of the rod in cms.,

d, b = the *depth* and *breadth* of a rectangular rod,

r = the radius of a rod of circular cross-section,

F = bending force in dynes,

e = bending of rod in cms.,

K, a constant, equal to 4 when the rod is fixed at one end and loaded at the other; and equal to $\frac{1}{4}$ when the rod is fixed at both ends and loaded at the middle,

then,* when the cross-sectional area is rectangular,

$$E = K\frac{FL^3}{bd^3e},$$

and, when the cross-sectional area is circular,

$$E = \tfrac{1}{3}K\frac{FL^3}{\pi r^4 e}.$$

Example.—Find the Young's modulus by flexure of a round steel bar supported at both ends.

$$L = 73 \text{ cms.,} \qquad F = 5000 \times 981 \text{ dynes,}$$

$$r = 0{\cdot}64 \text{ cms.,} \qquad e = 0{\cdot}17 \text{ cms.,}$$

$$E = \frac{5000 \times 981 \times 73^3}{12 \times 0{\cdot}17 \times \pi \times 0{\cdot}64^4}$$

$$= 1{\cdot}77 \times 10^{12} \text{ dynes per square cm.}$$

Exercises.

(1) Find the Young's modulus by flexure of a glass rod, supported at both ends; (2) also of the above steel rod when fixed at one end.

Experiment 61.—**To find the ratio between the lateral shrinking and the longitudinal extension of a stretched elastic body** (Poisson's ratio).

Instruments required.—A length of solid rubber, scale

For the proof of this see Jamin and Bouty, *Cours de Physique*, vol. i. part 2, p. 212 *et seq.*

pan, and weights, metre scale, beam compass, and micrometer screw-gauge.

When a wire or rod is stretched the area of its cross-section diminishes; the ratio of this lateral shrinking to the longitudinal extension was investigated by Poisson, and is often called Poisson's ratio. This ratio is not the same for all substances, as Poisson erroneously assumed; for india-rubber the ratio is $\frac{1}{2}$, for glass and metals it is between $\frac{1}{4}$ and $\frac{1}{2}$. As india-rubber undergoes great change of form under tension it is easiest to examine this substance. In making the experiment fix about a metre of solid rubber to a clamp, and attach a scale pan below, make two marks on the rubber, and take the distance between these marks, and also the diameter of the rubber at several marked points, when there is the least weight in the pan that will keep the rubber straight. Now add a greater weight and again take the length between the marks, and the diameter at the marked points; add successively other equal weights and repeat the measurements, noting down each time the weight and the corresponding length and diameter of the rubber, the latter being the mean of the diameters taken at the different points. The extension per unit length a is now found by dividing the observed extension under a given load by the whole length, and the corresponding lateral shrinking per unit of breadth β is found by dividing the observed shrinking by the whole breadth. Thus in the example which follows a cylindrical piece of solid rubber, with least weight to stretch it, was 789 mm. long, and the mean of five measurements

of its breadth was 7·67 mm. With a load of 500 grammes the length became 843 mm., and the breadth 7·45 mm. The extension was therefore $843 - 789 = 54$ mm., this divided by 789 gives $\alpha = 0·070$. The corresponding lateral shrinking was $7·67 - 7·45 = 0·22$ mm., this divided by 7·67 mm. gives $\beta = 0·029$. Dividing β by α we obtain the so-called Poisson's ratio σ. In this case it is found to be about $\frac{1}{2}$, that is to say the contraction β is equal to half the extension α. This is precisely the ratio that should be found if the total volume of the body remained unchanged, that is if it were incompressible, india-rubber must therefore be very nearly incompressible. That no change of the volume of the rubber occurs on stretching can be proved by fixing one end of an elastic band at the bottom of a narrow glass tube filled with water, and then, by means of a silk thread or fine wire attached to the rubber, stretching it; if the rubber remains wholly immersed no change in the level of the water should be seen.* When rubber is stretched it undergoes therefore a shearing strain (see p. 135).

Example.—Find Poisson's ratio for india-rubber.

Eight experiments were made, with a load increasing by 500 grammes in each experiment. The results were as follows :—

* See Jamin and Bouty, *Cours de Physique*, vol. i. part 2, p. 174 *et seq.*, also Tomlinson, *Phil. Trans.* 1881, and Mallock, *Proc. Roy. Soc.* 1879. Ayrton and Perry, by means of their spiral springs, have also determined the ratio σ for some metals (see *Proc. Royal Soc.*, 1884).

Number of Experiment.	Length in mm.	Diameter in mm.	α.	β.	$\sigma=\frac{\beta}{\alpha}$.
1	789	7·67	·070	·029	·41
2	843	7·45	·075	·033	·44
3	907	7·20	·093	·038	·41
4	992	6·92	·081	·043	·53
5	1073	6·62	·087	·040	·46
6	1167	6·35	·077	·039	·51
7	1257	6·10	·077	·041	·53
8	1354	5·85			

Mean = ·498.

Or say ·5 for increments of 500 grammes.

Exercise.

Determine the ratio σ for india-rubber of round and square section.

Experiment 62.—**Determination of the resilience of an elastic substance.**

Instruments required.—A wire, india-rubber strip, millimetre scale, scale pan, and weights.

The term *resilience* was introduced by Dr. Thomas Young to denote resistance to impulse, as distinguished from tenacity or strength, which denotes resistance to tension or pressure. "Resilience is jointly proportional to strength and toughness, and is measured [by the work that can be done upon the body, that is] by the product of the mass into [half] the square of the velocity of a body capable of breaking it, or of the mass and the height

from which it must fall in order to acquire that velocity, while strength is measured by the greatest pressure a body can support in a state of rest." * The resilience can be calculated if we know the tenacity and the extension the body undergoes before breaking. Thus if a body broke under the tension of 100 lbs. and extended 1 inch, the same weight would break it by striking it with a velocity acquired by falling from the height of half an inch, or a weight of 1 lb. would break it falling through 50 inches.†

Lord Kelvin (Sir W. Thomson) defines resilience as follows: "Resilience denotes the quantity of work that a spring (or elastic body) gives back when strained to some stated limit and then allowed to return to the condition in which it rests when free from stress. The word 'resilience' used without special qualification may be understood as meaning *extreme resilience*, or the work that a spring gives back when strained to the extreme limit within which it can be strained again and again without breaking or taking a permanent set.

"In all cases for which Hook's law of simple proportionality between stress and strain holds, the resilience is obviously equal to the work done by a constant force of half the amount of the extreme force, acting through a space equal to the extreme deflection.

* Young's *Lectures on Nat. Phil.*, delivered A.D. 1807, p. 111.

† Young reckons resilience as the work required to be spent upon a body in order to break it by impact, that is, $\frac{1}{2}Fs$, where s represents the total extension of the body at rupture; but what we now call resilience is the work a body can *give back* when strained. The extension, s, therefore must represent the maximum elongation *within the limits of elasticity*.

"When force is reckoned in gravitation measure, resilience per unit of the spring's mass is the height to which the spring itself or an equal weight could be lifted against gravity by an amount of work equal to that given back by the spring returning from the stressed condition." *

The longitudinal resilience of a substance can therefore be found if its Young's modulus be known, together with its density and the extreme elongation that the body undergoes within its elastic limit. India-rubber is characterised by its very high resilience, amounting to seven times that of steel wire and more than 230 times the longitudinal resilience of German silver.

(1) In determining longitudinal resilience, the wire, or substance under trial, is suspended from a rigid support, with a scale pan at the lower end in which to put the stretching weight, the elongation being noted by means of a suitable pointer and scale; the weight in the scale pan is gradually increased until the elastic limit is just overpassed, that is when on removing the weight from the scale pan the pointer does not return to its original zero. The elongation immediately preceding this is the one taken, ϵ, in the calculation. Then if

L = longitudinal resilience in cms.,

E = Young's modulus in dynes per square cm.,

e = total limiting elongation in cms.,

l = length of the wire in cms.,

ϵ = elongation per unit length $= e/l$,

* See Sir William Thomson's (Lord Kelvin's) *Mathematical and Physical Papers*, vol. iii. pp. 42, 43.

$A =$ cross-sectional area of the wire in sq. cms.,
$\rho =$ density of the wire,

$$E = \frac{Fl}{Ae} \text{ (see p. 142)}, \quad \therefore \ F = EAe/l = EA\epsilon.$$

And since the energy or work done in stretching a wire is equal to half the product of the stretching force into the elongation produced, the work is $= \frac{1}{2}EA\epsilon \times l\epsilon = \frac{1}{2}EAl\epsilon^2$ in kinetic units. But the work expended in raising a mass m of the same wire through a height L cms. in the same units is mgL; therefore $mgL = \frac{1}{2}EAl\epsilon^2$. Putting in the values of ϵ, and $m = lA\rho$, we obtain

$$L = \frac{Ee^2}{2\rho l^2 g}.$$

Example.—Find the longitudinal resilience of German silver wire.

$$l = 125 \text{ cms.}, \quad e = 0{\cdot}34 \text{ cms.}, \quad \rho = 8{\cdot}2.$$

By a separate experiment with this same wire Young's modulus was found to be $1{\cdot}092 \times 10^{12}$ dynes per square cm.

$$\therefore \ L = \frac{1{\cdot}092 \times 10^{12} \times 0{\cdot}34^2}{2 \times 8{\cdot}2 \times 125^2 \times 981} = 517 \text{ cms.}$$

(2) When the longitudinal extension and the lateral contraction of a body under stress are large, such as occurs with india-rubber, the above method of determining its resilience is less applicable. In this case the resilience is directly deduced from the work given back by the rubber when stretched within its elastic limit. In making the experiment the rubber is fixed to a support,

and a scale pan hung on the lower end; the weight in the pan is gradually increased up to the elastic limit of the rubber, the elongations produced by each additional weight being carefully noted. The results are then plotted, having weights (grammes) as abscissæ and elongations (cms.) as ordinates, and the area enclosed by the plotted curve represents the work which would be given back by the india-rubber if it had been released before it broke down. The work done, as represented by this area, divided by the weight of the india-rubber, will be the longitudinal resilience of the rubber.

Example.—Find the longitudinal resilience of an india-rubber band weighing 1·2 gramme.

Enter results thus—

Elongation in Cms.	Weight in Grammes.	Elongation in Cms.	Weight in Grammes.
2·0	300	38·0	2400
8·0	600	42·0	3000
15·2	900	56·5	4000*
22·1	1200	59·0	6100
27·6	1500	64·4	7100
31·8	1800	Broke	7500

The elastic limit occurred about here,* and by plotting the values so far we find that the enclosed area represents 108,600 cm. grammes;

$$\therefore \ L = \frac{108600}{1 \cdot 2} = 90500 \text{ cms.}$$

Note.—A thicker piece of solid vulcanised rubber, used in Experiment 61, and evidently of poorer quality, gave a much lower resilience. The student can easily test for himself the quality of different specimens of rubber bands or tubing (see Table XIII.).

Exercises.

Find the longitudinal resilience of (i.) brass wire, (ii.) copper wire, and (iii.) specimens of india-rubber.

Experiment 63.—**Determination of the torsional resilience of a spiral spring.**

Instruments required.—A spiral spring, a centimetre scale, a scale pan, and gramme weights.

Take a length of wire and make it into a spring by means of a mandril and hand-brace; fix one end of the spring to a firm support—the spring hanging vertically downwards—and attach a scale pan to the other end in which to put the stretching weights.

A suitable pointer is attached to the lower end of the spring or to the scale pan which indicates on a centimetre scale the extension of the spring for a given weight.

In making the experiment put a weight on the scale pan and note the elongation, then take off the weight and observe if the pointer returns to the zero. Now put more weights on the pan, note the elongation, and again remove the weights, observing if the pointer returns to zero. Do the same thing for each increase of weight in the pan until, on taking all the weights off

the pan, the pointer does not return to the zero, when this happens the elastic limits of the spring have been just exceeded.

The weight which was on the scale pan immediately before this limit was reached is the weight to be used in the calculation. If

T = torsional resilience of the spring,

W = weight in grammes in the scale pan just before the elastic limit is reached,

m = weight of scale pan in grammes,

m' = weight of the spring in grammes,

e = elongation in cms. of the spring produced by the weight W,

then
$$T = \frac{(W + m)e}{2m'}.$$

Example.—A piece of German silver wire was made into a spiral spring, and the resilience obtained.

$$m = 1{\cdot}02 \text{ grammes}, \quad m' = 0{\cdot}65 \text{ grammes}.$$

Weight in the Scale Pan in Grms. W.	Reading on the Scale in Cms. (Zero of scale = 35·05.)	Elongation of the Spring in Cms. *e*.
10	32·75	2·30
15	31·25	3·80
20	29·20	5·85
25	26·67	8·38
30	22·60	Exceeded limits

Then
$$T = \frac{(25 + 1{\cdot}02)8{\cdot}38}{2 \times 0{\cdot}65} = 167{\cdot}7 \text{ cms.}$$

Or the torsional resilience of the spring is equal to the work required to lift its weight through nearly 168 cms. (see Table XIII.).

Note.—The longitudinal resilience is

$$L = \frac{2TM\epsilon^2}{n\delta^2}, \text{ where}$$

T = torsional resilience,
M = Young's modulus,
n = rigidity modulus,
ϵ = extreme elastic elongation,
δ = extreme elastic strain.

Exercise.

Make springs of steel, platinoid, German silver, and copper, and determine the resilience of each.

CHAPTER VIII

MECHANICAL PROPERTIES OF SOLIDS (*Continued*)

Rigidity—Viscosity—Friction

THE rigidity or stiffness of a solid has already been defined. This property of solids can be determined by measuring the distortion which a body undergoes without accompanying change of volume, this has already been defined as a *shear* (p. 135). The ratio of the force per unit area, producing the strain, to the strain produced is, as in Young's modulus, the coefficient of simple rigidity; only in this case the strain is a shear. It is convenient in determining this coefficient to have the body in the form of a wire or thread, and to apply the force by means of a twist given to the lower end of a wire hung vertically. As already stated (p. 135) the couple required to twist one end of a wire of unit length through unit angle, the other end being fixed, is called the modulus of torsion M of the wire: if the angle be θ, and the length be l, then the couple is $M\theta/l$.

Unit angle or RADIAN is the angle subtended at the centre of a circle by an arc equal in length to the radius $= 57^{\circ}{\cdot}29$.

Experiment 64.—**To determine the modulus of torsion of a wire.**

Instruments required.—Firm clamp, a wire with weight at lower end to vibrate, and a stop-watch.

(1) To find the modulus of torsion experimentally, the wire is firmly fixed at one end to a support with a mass of known moment of inertia attached to its lower end; the wire is twisted and allowed to oscillate, the time of oscillation t_1 being taken.

(2) If the vibrator be of irregular shape, t_1 is taken as before, then a body of known moment of inertia is added (p. 133), and the time of oscillation t_2 again taken.

If τ = modulus of torsion,

l = length of the wire in cms.,

I = moment of inertia of vibrator,

I′ = „ „ added vibrator,

t_1 = period in seconds of first vibrator,

t_2 = period in seconds with I altered,

then $$\tau = \frac{4\pi^2 l\mathrm{I}}{t_1^2} \text{ in (1), and } = \frac{4\pi^2 l\mathrm{I}'}{t_2^2 - t_1^2} \text{ in (2).}$$

This gives the force of the couple required to twist unit length of the particular wire under experiment through unit angle: to find the modulus of torsion of the material of which the wire is made, this value of τ must be divided by $\frac{1}{2}\pi r^4$, where r is the radius of the wire in cms.

Example.—Find the modulus of torsion of a German silver wire.

The vibrator was a right cylinder with the axis of vibration the axis of figure, and the moment of inertia altered by increasing the length of the cylinder.

$$I' = M\frac{R^2}{2} \text{ (see Table XIV.)},$$

$$I' = 6912 \times \frac{5^2}{2} = 86400 \text{ cm. grams.},$$

$$l = 187 \text{ cms.}, \quad t_1 = 8{\cdot}3 \text{ secs.}, \quad t_2 = 9{\cdot}2 \text{ secs.},$$

$$\therefore \quad \tau = \frac{4\pi^2 \times 86400 \times 187}{9{\cdot}2^2 - 8{\cdot}3^2} = 4{\cdot}05 \times 10^7.$$

Exercise.

Find the modulus of torsion of an iron wire; and also of a copper wire.

Note.—The modulus both of elasticity and torsion varies with the temperature. Tomlinson has shown that a considerable and permanent loss of ductility and a remarkable increase of elasticity is produced in annealed iron, and the latter also in steel, by raising the temperature to 100° C.; but a temporary loss of elasticity occurs in nickel, copper, etc., by heating to the same temperature. In the case of iron, steel, nickel, and probably cobalt wire, the modulus undergoes a profound change at a temperature corresponding to the loss of the magnetic properties of these metals.

Experiment 65.—**To determine the simple rigidity of a wire.**

Instruments required.—The same as in the last experiment.

The wire is fixed at one end to a clamp, and hung vertically with a vibrator firmly attached to the other end; the vibrator, being free, is made to oscillate round

the vertical axis of the wire as its axis, and the period of oscillation taken. If

n = the simple rigidity,
l = length of the wire in cms.,
r = radius of the wire in cms.,
I = moment of inertia of the vibrator in cm. grammes,
t = time of oscillation, *i.e.* half period in secs.,
$g = 981$, $\pi = 3{\cdot}1416$, then

$$n = \frac{2\pi \mathrm{I} l}{g t^2 r^4}.\text{*}$$

Example.—Find the simple rigidity of a German silver wire.

The vibrator is a right cylinder with the axis of vibration the axis of the figure $\mathrm{I} = \mathrm{M}\frac{\mathrm{R}^2}{2}$ (see Table XIV.).

$$\mathrm{M} = 6912 \text{ grammes}, \quad \mathrm{R} = 5 \text{ cms.},$$

$$\mathrm{I} = 6912 \times \frac{5^2}{2} = 86400 \text{ cm. grammes},$$

$$l = 187 \text{ cms.}, \quad r = {\cdot}0625 \text{ cms.}, \quad t = 4{\cdot}2 \text{ cms.},$$

$$n = \frac{2\pi \times 86400 \times 187}{981 \times 4{\cdot}2^2 \times {\cdot}0625^4}$$

$$= 3{\cdot}85 \times 10^8 \text{ grams. per square cm.}$$

Exercise.

Find the rigidity modulus of a steel wire, and also of a brass wire.

* See Appendix, § 15, and Sir W. Thomson's *Mathematical and Physical Papers*, vol. iii. p. 78.

Experiment 66.—**To determine the coefficient of simple rigidity by Maxwell's vibration needle.**

Instruments required.—Same as in Experiment 64.

Maxwell's vibration needle consists of a hollow tube twenty or more cms. in length, with a binding screw projecting perpendicularly from its middle, to which the lower end of the wire to be tested is attached. There are four other tubes, each *one-quarter* the length of the needle, which closely fit inside. Two of these tubes are hollow, and the other two filled with lead.

In making the experiment the wire is firmly fixed in a clamp at one end, and the needle fixed at the other end, with the four tubes inside the needle. The two small loaded tubes being first put in the middle, the period of vibration is observed. The position of the small tubes is now reversed, the two hollow tubes being put in the middle and the loaded ones outside, and the period again observed. Then if

$n =$ the coefficient of simple rigidity,
$l =$ length of the wire in cms.,
$r =$ radius of the wire in cms.,
$c =$ length of the needle,
$m_1 =$ weight in grammes of a hollow tube,
$m_2 =$ weight in grammes of a loaded tube,
$t_1 =$ period when loaded tubes in middle,
$t_2 =$ period when hollow tubes in middle,

$$n = \frac{2\pi l c^2 (m_2 - m_1)}{g r^4 (t_2{}^2 - t_1{}^2)}.*$$

* See Gray's *Absolute Measurements*, vol. i. chap. iv. § 3.

Example.—Find the coefficient of simple rigidity of a brass wire.

$$l = 144{\cdot}2 \text{ cms.}, \quad r = {\cdot}0162 \text{ cms.}, \quad c = 19{\cdot}4 \text{ cms.},$$
$$m_1 = 11{\cdot}75 \text{ grams.}, \quad m_2 = 34{\cdot}67 \text{ grams.},$$
$$t_1 = 25{\cdot}8 \text{ secs.}, \qquad t_2 = 32{\cdot}0 \text{ secs};$$

$$\therefore n = \frac{2\pi \times 19{\cdot}4^2 \times 144{\cdot}2 \times 22{\cdot}92}{981 \times {\cdot}0162^4 \times 55{\cdot}8 \times 6{\cdot}2}$$
$$= 3{\cdot}34 \times 10^8 \text{ grams. per square cm.}$$

Note.—$(t_2^2 - t_1^2) = (t_2 + t_1)(t_2 - t_1) = 55{\cdot}8 \times 6{\cdot}2.$

Exercise.

Find the coefficient of simple rigidity of a fine steel wire, or of a fine German silver wire by Maxwell's vibration needle.

Experiment 67.—**The viscosity of solids.**

Instruments required.—As described below.

Solids, even such as iron and steel, are to some extent viscous, that is, their resistance to change of shape depends more or less on the *time* to which stress has been applied to them. Also cobbler's wax and ice, which, under sudden stress, are very brittle substances, are really viscous, and if time be allowed will flow down an incline. An instructive model of a glacier can easily be made of cobbler's wax and its motion studied.

The viscosity of metals has been illustrated by the experiments of Tresca, who by means of enormous pressure caused metals, such as copper and iron, when cold,

to "flow" through a hole in a disc much as a viscous liquid would behave.

The following experiments with a wire of soft iron, or better, aluminium, will illustrate this property in metals.

Take the wire and fix one end of it to a firm support, suspend a small weight on the other end, the wire hanging vertically downwards. Attach firmly to the lower end a pointer moving over a graduated circle, so that the wire can be twisted through any given angle, the force of torsion being proportional to this angle. Also fasten a small mirror to the lower end of the wire, to indicate the motion of the wire when twisted; this motion can be observed by a telescope and scale, or a beam of light from the mirror falling on a scale.

(1) Note the position of rest of the spot of light reflected from the mirror, then twist the wire through a given angle and at once release it, allowing it to return gently toward the zero *without oscillation.* It will be found that the spot does not return exactly to its original zero. Next keep the lower end of the wire twisted through the same angle for some minutes, the displacement from the first zero, when the wire is gently allowed to come back, will be increased, and so on. It will be observed that the spot will slowly creep back nearer to its original position of rest when the wire is left undisturbed.*

* The probable explanation was given by Wiedemann: as the wire vibrates torsionally the molecules likewise rotate about their axes, and at each rotation are permanently deflected, so that if at the end of one swing the vibrator be checked and gently restored towards zero, a deflection remains. If, however, the vibration be allowed to continue, the loss of

(2) Twist the wire *in one direction* through a given angle and keep it so for thirty minutes, then twist it through the same angle in the opposite direction for sixty minutes, and then in the first direction again for ten minutes, and note that in recovering itself the wire will go through an opposite series of twists, its position of displacement remaining for a longer or shorter time in the one direction or the other according to the length of the period during which it has been previously twisted.

(3) Heat the wire to redness whilst slightly stretched; this will restore it to a virgin condition. Now make a series of experiments, twisting the wire through an increasing angle each time and gently releasing it, noting each time the displacement from zero of the spot of light. The permanent set or twist will be found to increase as the force of torsion increases. Plot the results in a curve, taking for abscissæ the torsional couple given to the wire, and as ordinates the permanent set; the curve will be found very similar to the curve of brittleness (Fig. 47).

Sir W. Thomson has found that when a wire had been kept oscillating for days continuously it appeared to undergo an "elastic fatigue," so that if stopped for a moment and then made to vibrate, its arc of vibration fell to one half in less than half the time taken by an unfatigued wire; the fatigued wire was, however, found to recover itself after a Sunday's rest.

energy which is due to internal friction is caused by the work done in alternately twisting the molecules from their permanent set on one side to a corresponding position on the other side. See Tomlinson's Memoirs, *Phil. Trans.* 1888, and *Proc. Physical Society*, 1887.

Tomlinson * has noticed a similar effect on the value of Young's modulus; upon heavy loading and unloading an increase of elasticity takes place after an interval of rest. With some metals, such as aluminium and zinc, complete recovery on the removal of stress is only attained after several hours. As already mentioned, p. 136, Bottomley has found a similar effect in the tenacity of iron wire.

These elastic "after-effects" are all due to viscosity, which even the most rigid solids possess in some degree, and the relative value of which can be found by the *logarithmic decrement* of an arc of oscillation (see Experiment 81). Maxwell † has suggested that these effects are due to the behaviour of different groups of molecules in the solid, some are in a more stable condition than others, and therefore retain longer their original configuration.

Experiment 68.—**To find the hardness of a solid.**

Instruments required.—Specimens of minerals and metals and a file.

There are certain familiar properties of solids, of which hardness is an example, which have not as yet received sufficient investigation from a physical standpoint ; exact definitions are wanting and absolute measurements cannot be made. Hardness is usually defined as the resistance which a solid offers to being cut or *scratched,* the hardness of a metal is the resistance it offers to the file or to an engraver's tool; it does not

* *Phil. Trans.* 1881 *et seq.* The student who wishes for more information on the subject of the viscosity of metals should refer to Mr. Tomlinson's numerous and laborious experiments.

† *Ency. Brit.* 9th ed., Art. "Constitution of Bodies."

necessarily measure the resistance of a metal to *abrasion* by friction. Thus a metal like aluminium bronze is softer than (that is can be scratched by) steel, yet it resists abrasion better than steel; needles employed for perforating stamps made of aluminium bronze last longer than steel needles.

Only a relative estimate of the hardness of bodies is possible at present.* Mineralogists use a scale of hardness, consisting of ten minerals, which gradually increase in hardness from 1 to 10. This scale is given in Table XVI., each mineral in this series scratches the one above or is scratched by the one below it. In this way a comparative estimate of the hardness of a body can be formed. Thus if a body scratches fluorspar but is scratched by apatite, its hardness H is expressed as 4·5. Or if a light file is found to scratch the body under trial with the same ease as it scratches fluorspar, the body has a hardness of 4. Only about a dozen minerals, including the precious stones, are harder than flint, H = 7. Among metals the order of hardness is lead, which is below 2·5; tin, cadmium, bismuth, silver, gold, copper, platinum, palladium, which are below 3; zinc, antimony, cobalt, nickel, iron, which are above 3 but below 5·5; steel and manganese steel, which are from 6 to 7. The alloys are harder and have higher tenacity than the pure metals from which they are made. An alloy of steel with 15 per cent of manganese, discovered and made by Mr. R. A. Hadfield, an eminent steel manufacturer of Sheffield,

* Hertz and Auerbach have recently sought to give a more precise definition to and estimate of hardness.—*Wied. Ann.* vol. xliii.

possesses extraordinary hardness and toughness combined, besides other remarkable properties, such as very high elasticity, extremely high resilience, and though 85 per cent is iron can neither be magnetised nor is it a magnetic body.

Attempts have been made to obtain a more perfect measure of hardness by measuring the abrasion caused by a drill actuated by a known force, but, as already stated, this is not quite the same as hardness. Moreover, the rate at which a drill, a disc, or loose particles are moving alters their apparent hardness; a disc of iron spinning at a high velocity can cut steel or agate, though easily cut by these bodies if it spin slowly.

The best arrangement for determining *the coefficient of abrasion* is that devised by Dr. Trouton of Trinity College, Dublin. It consists of two cylinders of the substance to be tested, which are made to revolve and abrade each other by uniform pressure, the amount of material abraded in a given time is then weighed. The coefficient of abrasion ϵ may be defined as the reciprocal of the weight of material abraded in unit time from a body of unit area moving with unit velocity, like substances being forced together by unit pressure. If then the area of the surface be A, the pressure P, the velocity V, and time τ, the weight of material abraded, W, will be $\mathrm{PAV}\tau/\epsilon$, or

$$\epsilon = \frac{\mathrm{PAV}\tau}{\mathrm{W}}.$$

As $\mathrm{P} = \mathrm{F}/\mathrm{A}$, the area of the rubbing surfaces may be neglected.* The coefficient of abrasion as thus defined

* See *Brit. Assoc.*, Report 1890.

affords an *absolute* as distinguished from a purely arbitrary measure of hardness: though the method is more suitable for rocks than metallic surfaces.

Heat profoundly affects the hardness of bodies, and at melting most of the metals pass somewhat abruptly to the liquid state; iron and platinum pass through an intermediate viscous condition, enabling them to be *welded.* The presence of a small percentage of carbon in iron converts it into steel, and reverses the ordinary annealing process (footnote, p. 137), so that heating and sudden cooling give to steel a high degree of hardness or "temper."

The *plasticity* of a metal is not the same as its softness, lead is softer than silver but less plastic. Plasticity includes both malleability and ductility, and may be defined as susceptibility to change of form (such as extension of surface or of length) without rupture (see p. 140). Here again the most malleable of metals, gold, is by no means the most ductile, platinum, silver, iron, and copper coming before it.

Exercise.

Find the hardness of a piece of steel before and after tempering.

Experiment 69.—**To determine the coefficient of friction.**

Instruments required.—A plane surface with pulley, a scale pan, and weights.

In Fig. 49 T is a table on which is fixed the surface S to be tested against the slider R, which carries a weight

M. A cord attached to R passes over a pulley P, and has a scale pan at the other end which serves to hold the applied weight m.

The initial cohesion or "stiction" of the surfaces is so uncertain that in making the experiments the weight M

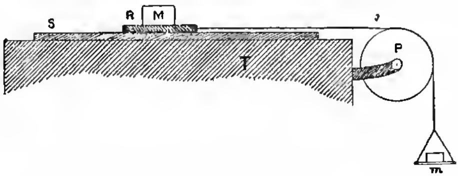


Fig. 49.

is given a start and the weight m so adjusted that M with the slider R will move along the plane S with a uniform motion.

Several experiments are made for different values of M and m, and the coefficient, which is the ratio of the two weights, is calculated for each and the mean value taken.

In accurate experiments a correction must be made for the friction of the wheel, as is done in Atwood's machine (see p. 115). If

$\mu =$ the coefficient of friction,

$W =$ weight of the slider and mass M,

$w =$ weight of the scale pan and mass m,

then $$\mu = \frac{w}{W}.$$

Example.—Find the coefficient of friction between a surface of oak and a slider of oak with the grain crosswise.

Enter results thus:—

Number of Experiment.	W Grammes.	w Grammes.	μ.
1	1200	672	·56
2	2400	1292	·53
3	3600	1944	·54
4	4800	2448	·51
5	6000	3000	·50

Mean = 0·528.

Exercises.

Find the coefficient of friction—

1. Pine on pine (parallel).
2. Pine on pine (crosswise).
3. Oak on oak (parallel).
4. Cast iron on pine with and without a lubricant.

Experiment 70.—**To determine the angle of friction.**

Instruments required.—An inclined plane with an arc for measuring the angle of inclination, a slider, and weights.

In Fig. 50 L is the inclined plane, which can be set to any angle, and the angle measured by the circular arc D. The experimental surface S is fixed to the inclined plane, and over this moves a slider carrying a weight M, as in the last experiment.

In making the experiment the inclined plane is slowly raised, the weight with slider is given, as before, an initial start, and the particular angle of inclination noted, when

the weight M with slider continuously slides down the surface.

Different weights on the slider R are now tried, and

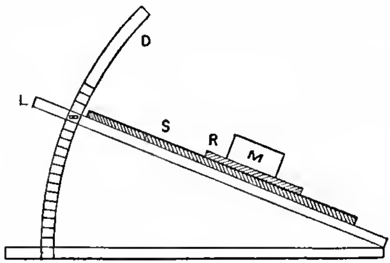


Fig. 50.

the angle with each noted, which will not be found to vary very much. If

$$\theta = \text{mean angle of inclination,}$$
$$\mu = \text{coefficient of friction,}$$

then $$\mu = \tan\ \theta.$$

Example.—Find the angle of friction and the coefficient of friction of oak on oak crosswise.

Enter results thus:—

Number of Experiment.	Total Load on Slide in Grammes.	θ.	Tan θ or μ.
1	1200	27°0′	·51
2	2400	29°14′	·56
3	3600	26°32′	·50
4	4800	27°50′	·53
5	6000	29°36′	·57

Mean = 28°2′ 0·534.

Exercises.

Find the angle of friction and the coefficient of friction of—

1. Pine on pine (parallel).
2. Pine on pine (crosswise).
3. Oak on pine.
4. Iron on pine with and without lubricants.

Additional Exercises on Friction.

1. Put a string over a smooth fixed peg, hang a weight W at one end of the string, and load the other end until motion occurs.

2. Now lap the string once round the peg and so increase their surface of contact, keeping W the same, find the new load required to move W. Repeat the experiment with two or more laps.

3. Find the value of the friction in Atwood's machine with different moving weights.

CHAPTER IX

MECHANICAL PROPERTIES OF LIQUIDS

A FLUID, whether liquid or gaseous, is a body which possesses no elasticity of shape; that is to say, a fluid is a body which requires no force to keep it in any particular shape. A fluid therefore offers no permanent resistance to change of shape. Owing to viscosity fluids do offer a temporary resistance, but this is entirely different from the *permanent* resistance to change of shape which is the characteristic of a solid. In fluids the force of resistance to change of shape is in simple proportion to the velocity of the change. This resistance is due to molecular friction in the fluid, and the greater the resistance the more viscous the fluid is said to be. From Experiment 67 in the last chapter, we have seen that solids, in addition to their *elastic* resistance, also possess a *frictional* resistance against change of shape; the viscosity of solids, like the viscosity of fluids, is therefore due to their internal or molecular friction.

It will be seen from the foregoing definition of a fluid that *elasticity of volume* is the only elastic coefficient possessed by liquids and gases. Boyle's law expresses

this coefficient for gases, and holds true within wide limits for all the permanent gases. Liquids, on the other hand, have different coefficients of elasticity, according to the nature and temperature of the liquid, and for very high pressures the coefficient also varies.

Whilst the limit of elasticity of most solids lies within a narrow range, that of fluids is practically unlimited. Gaseous fluids are characterised by their indefinite expansion (p. 92), and in general differ from liquids and solids, by their enormously greater proportional change of volume with change of pressure. We shall show first how the coefficient of elasticity of a liquid is measured, and then examine some other of the mechanical properties of liquids. If

$V =$ the original volume of the liquid,

$P =$ the pressure per unit area,

$v =$ the diminution of volume due to the stress P,

then $\frac{v}{V} =$ the strain per unit volume, and the coefficient of elasticity of volume $= \frac{\text{stress}}{\text{strain}}$ is $P/\frac{v}{V} = \frac{PV}{v}$.

The reciprocal of this coefficient is known as the *compressibility*, viz. $\frac{v}{PV}$.

Experiment 71.—**To measure the compressibility of a liquid.**

Instrument required.—A piezometer.

The piezometer (pressure measurer) is an instrument devised by Oersted for measuring the coefficient of

compressibility of liquids. It consists of a strong cylindrical vessel of glass filled with water (Fig. 51), fitted with a screw-plunger working through a leather collar. The liquid whose compressibility is to be determined is caused to fill a bulb having a capillary tube, in which is an index of mercury. It is better to invert the bulb so that the capillary tube dips into a little vessel of mercury within the piezometer; by slightly warming the piezometer bulb and allowing it to cool with the open end dipping in the mercury the latter can be made to rise in the tube to a convenient height. To fill the piezometer bulb it must be warmed and a little of the liquid allowed to enter; by boiling this gently, and inverting the open end in the liquid, the tube can be readily filled in two or three operations. To measure the pressure a glass tube closed at one end and open below, and as long as possible, is employed; the observed change of volume of the air within the tube enables the pressure to be calculated from Boyle's law. It is necessary to calibrate this pressure tube beforehand and graduate it, or fasten it against a millimetre scale; the capillary tube of the piezometer should also be calibrated, though a small portion of its length only will be required. It is also necessary that a constant temperature should be maintained throughout the experiment. The temperature of the air is most

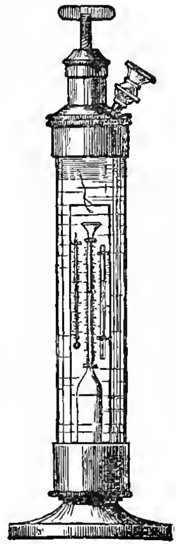


Fig. 51.

convenient, and in this case the piezometer should be filled the day before the actual experiment is made.

In making the experiment, first weigh the empty dry bulb. Then by a thread of mercury of measured length find the radius of the tube (Experiment 17), and determine the ratio of the content of one division of the capillary tube to the whole content of the bulb and then up to the mercury index. Next fill the bulb with distilled water at a known temperature, and weigh again; this enables the volume of the bulb to be found. If water be the liquid whose compressibility is to be found, the experiment can now be made, otherwise the bulb must be emptied by boiling, dried, and refilled with the liquid to be tried. When all is ready, and the cap of the instrument screwed down, make a series of observations till a pressure of about three or four atmospheres has been indicated; beyond this the glass cylinder may give way. Next deduce the atmospheric pressure corresponding to the values of the manometer readings, and calculate the corresponding compressibility. To the results found, the percentage compressibility of glass must be *added*. The reason for this is that owing to the hydrostatic pressure the interior content of the piezometer bulb has been diminished just as if it had been a solid piece of glass by an amount corresponding to the compressibility of the glass.

Calling V the capacity of the piezometer bulb, P the pressure, μ the coefficient of compressibility of the liquid, the diminution of volume is μPV. If the vessel were incompressible this would be observed, but it is not so;

let k be the cubical coefficient of compressibility of the vessel, its capacity will diminish to kPV, and let the observed contraction of the liquid in the piezometer be v (*best observed by means of the reading microscope*),

then $$v = \mu \mathrm{PV} - k\mathrm{PV},$$

whence $$\frac{v}{\mathrm{PV}} = \mu - k,$$

the quantity $\mu - k$ is observed, and hence to find μ we must *add* the cubical coefficient of compressibility of glass $k = 0{\cdot}0000025$ per atmosphere.

Example.—Find the coefficient of compressibility of water at 14° C.

(1) A length of 6·2 cms. of pure mercury at 14° C. was introduced in the capillary tube of the piezometer. The weight of this mercury was 0·671 gramme. Hence the *volume of the tube per cm.* of its length = 0·00798 c.c.

(2) The weight of the dry bulb having been taken, the weight of water filling it at 14° C. up to a zero mark was 38·052 grammes. The density of water at 14° C. = 0·9993, hence the *volume of the bulb* = 38·052 ÷ ·9993 = 38·078 c.c., or corrected for the change of zero = 38·044 = V.

(3) The pressure of the atmosphere was 77·2 cms.; this, added to the column of water in the piezometer vessel = 1·0845 megadynes.

(4) Pressure was now applied, and from the diminution in volume of the pressure gauge was found to be = 4·363 megadynes, therefore the pressure added was 4·363 − 1·084 = 3·279 megadynes = P.

(5) Under this pressure the decrease in volume of the liquid was $\cdot 00559 = v$. Hence

$$\mu - k = \frac{\cdot 00559}{3{\cdot}279 \times 38{\cdot}044} = 0{\cdot}0000448$$

per megadyne per sq. cm., ∴ per atmosphere (p. 93),

$$\mu = \cdot 0000454 + \cdot 0000025 = 4{\cdot}79 \times 10^{-5} \text{ at } 14° \text{ C.}$$

Exercise.

Find the coefficient of compressibility of alcohol or sea-water.

Experiment 72.—**Determination of the work done by hydraulic machines.**

Instruments required.—Various kinds of water-pumps, Bramah press, and a water-ram.

(1) Examine the valves and principle of a lift-pump, force-pump, and also the combined lift and force-pump. Determine the work done during a single stroke when the water is flowing by catching the water in a beaker and weighing it, and measuring the height which the water in the pump-barrel has been raised by the stroke.

(2) Examine in the same way the valves and air chambers in the water-ram, and determine the mechanical advantage of the ram for various heights of head; *i.e.* find the ratio of the volume of water raised to the volume supplied by the feed cistern, the head being constant during an experiment.

(3) Examine also the hydrostatic bellows and the Bramah press, and determine the mechanical advantage of the press; that is, find the ratio of the areas of the pistons, the leverage employed, and the pressure as indicated by a pressure gauge.

Experiment 73.—**To determine the coefficient of efflux of a liquid.**

Instruments required.—A reservoir with ball-cock or supply pipe and overflow, beaker, and a balance.

When a liquid flows from an opeuing in a thin plate the quantity which flows out in a given time is *less* than that given by Torricelli's theory, on account of the contraction of the liquid vein ("vena contracta") immediately after leaving the orifice.* The observed volume divided by the calculated volume is called the coefficient of efflux.

Take a convenient vessel with an aperture in its side, closed by a thin metal plate with an orifice; this can be stopped by a plug that can be easily removed: keep the level of the liquid in the reservoir always constant; this can be arranged by a preliminary experiment.

In making the experiment open the orifice and allow the liquid to run into a beaker for a certain time, which may be noted by a stop-watch.

Weigh the quantity of the liquid in grammes, and knowing its temperature and density calculate the volume in c.c. Then if

K = the coefficient of efflux,
V = observed volume run out,
V' = the theoretical volume,
t = time of flow of liquid,
h = distance from the surface of the liquid to the centre of the orifice,
A = sectional area of orifice,
v = velocity of efflux.

* See Deschanel's *Physics*, edited by Everett, p. 225.

By Torricelli's theorem (see p. 190)

$$v = \sqrt{2gh},$$

$$\therefore\ V' = Atv = At\sqrt{2gh},$$

and
$$K = \frac{V}{V'}.$$

Example.—Find the coefficient of efflux of water through an orifice 7·5 mm. diameter. Since

$$h = 24{\cdot}6 \text{ cms.},\quad t = 30 \text{ secs.},\quad A = \pi \times {\cdot}375^2 \text{ sq. cms.},$$

$$\therefore V' = \pi \times {\cdot}375^2 \times 30(2 \times 981 \times 24{\cdot}6)^{\frac{1}{2}} = 2911 \text{ c.c.}$$

The weight of water which flowed out in 30 secs. = 1902 grammes at 8° C., hence

$$V = 1902 \text{ c.c.}$$

$$\therefore\ K = \frac{1902}{2911} = 0{\cdot}653.$$

Exercise.

Repeat the above experiment, using different orifices and different heads of water.

Experiment 74.—**Proof of Torricelli's law that the velocity of efflux of a liquid is proportional to the square root of the height of the head.**

Instruments required.—A tall vessel with lateral orifices, a balance and weights, a beaker, and a stop-watch.

With the arrangements for keeping the head constant, as in Experiment 73, open the lower orifice and receive in a beaker the liquid which flows out in, say, one minute, and determine the weight of the water. Repeat

the experiment with orifices at other distances below the head. If

W = grams. of water which flow out,
h = depth of orifice below the surface,
v = actual velocity of efflux in cms. per sec.,
v_1 = theoretical velocity of efflux,
A = area of orifice in sq. cms.,
t = time of flow in secs.,
K = the coefficient of efflux,

then
$$v = \frac{W}{At},$$

i.e. v is proportional to W, and

$$v_1 = \sqrt{2gh},$$

but
$$K = \frac{v}{v_1},$$

$$\therefore v = K\sqrt{2gh},$$

hence $W \propto \sqrt{h}$, or $\frac{W^2}{h}$ is a constant.

Example.—Make the above experiment with a vessel containing water, the orifice being of the same area.

Enter results thus:—

h Cms.	W Grams.	$\frac{W^2}{h}$.
84	708	5967
25	385	5930

Exercise.

Repeat the above experiment with various sizes of orifice.

Note.—It will be observed that the *density* of a liquid does not affect its rate of flow (see Experiment 79).

The law of Torricelli does not strictly hold if the orifice is at the end of a pipe or tap instead of being in the thin wall of the vessel, owing to resistance of the pipe, as the following results of an experiment show.

There were three taps in the side of a tall vessel at different heights, one nozzle fitted all the taps, so that the area of the orifice was the same in all cases.

h Cms.	W Grams.	$\frac{W^2}{h}$.
86	103	123·3
50·5	76	114·4
15	38	96·3

Experiment 75.—**To determine the height of a head of water by efflux.**

Instruments required.—The same as in Experiment 73.

A pipe leading from the bottom of a tall reservoir of water, or from a cistern of known height, has an orifice of known area: receive in a beaker the water flowing from the orifice in a given time, and from the weight of water calculate the velocity, thence deduce the height of the

head. Experiment gave the value of K as 0·73, owing to the nature of the orifice. If

W = weight of water in grams. which flows out in t secs.,
t = time of flow in secs.,
A = area of orifice in sq. cms.,
h = height in cms.,

then, from p. 179, $\sqrt{h} = \dfrac{W}{KAt(2g)^{\frac{1}{2}}}$.

Example.—Find the height of the water surface in a cistern in the laboratory.

$$A = \pi r^2 = \pi \times \cdot 153^2,$$

$$W = 1260 \text{ grams.,} \quad t = 30 \text{ secs.,}$$

$$\therefore \sqrt{h} = \frac{1260}{0{\cdot}73 \times \pi \times \cdot 153^2 \times 30(2 \times 981)^{\frac{1}{2}}},$$

$$\therefore h = 312{\cdot}2 \text{ cms.}$$

The height of the surface of the water in the cistern was by direct measurement found to be 310·8 cms.

Exercises.

Repeat the above experiment with different sizes of orifice. Care must be taken that the orifice is small relatively to the pipe.

Experiment 76.—**Comparison of two methods for determining the parabola formed by a spouting jet.**

Instruments required.—A vessel of water with a small orifice at the side, a beaker, and a millimetre scale.

(1) Take a tall vessel of water (Fig. 52), with an orifice O in the side near the bottom, which may be closed temporarily by a plug, and as before keep the head of

water constant. Place the vessel so that the distance AB = OA, and set the experiment going by removing the plug from O.

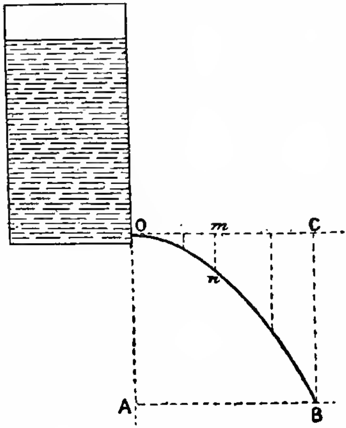


Fig. 52.

Now measure from the level OC the distance *mn* to the jet of water, say for every 10 cms. of OC, and plot a curve with OC as axis of abscissæ, and OA as axis of ordinates.

Example 1.—Find the curve for a water jet issuing from a lateral orifice. In the experiment the distances from the orifice are given in column x below, and the fall of the curve from the level of the orifice in column y.

The distance OA = AB = 70 cms.

x Cms.	y Cms.
0	0
10	1·2
20	4·9
30	10·0
40	18·0
50	32·5
60	44·0
70	68·0

By plotting these values we get the curve A in Fig. 53.

(2) Find the volume of water flowing from the orifice in unit of time, by receiving it in a beaker and weighing and calculating the volume. Let

W = weight of water run out in t seconds,
A = area of orifice,
v = velocity of efflux,

$$\therefore\ v = \frac{W}{\rho A t}.$$

Also, from dynamical considerations,*

$$x = vt, \quad y = \tfrac{1}{2}gt^2;$$

$\therefore\ y = \dfrac{g}{2v^2}x^2$, which is the equation to the parabola.

Then, knowing v, plot the curve whose equation is

$$y = \frac{g}{2v^2}x^2.$$

Example 2. It was found that

W = 99 grams. in 60 secs.,
$A = \pi \times {\cdot}0538^2$ sq. cms., $\rho = 1$,

$$\therefore\ v = \frac{99}{\pi \times {\cdot}0538^2 \times 60} = 181{\cdot}5 \text{ cms. per sec.,}$$

and $$y = \frac{981}{2 \times 181{\cdot}5^2}x^2 = {\cdot}0149x^2.$$

Then by giving various values to x in the equation $y = {\cdot}0149x^2$, we get the following table :—

* Garnett's *Dynamics*, chap. iii., or Wormell's *Dynamics*, p. 40.

x.	y.
0	0
10	1·49
20	5·96
30	13·41
40	23·84
50	37·25
60	53·64
70	73·01

By plotting these values we get the curve B in Fig. 53.*

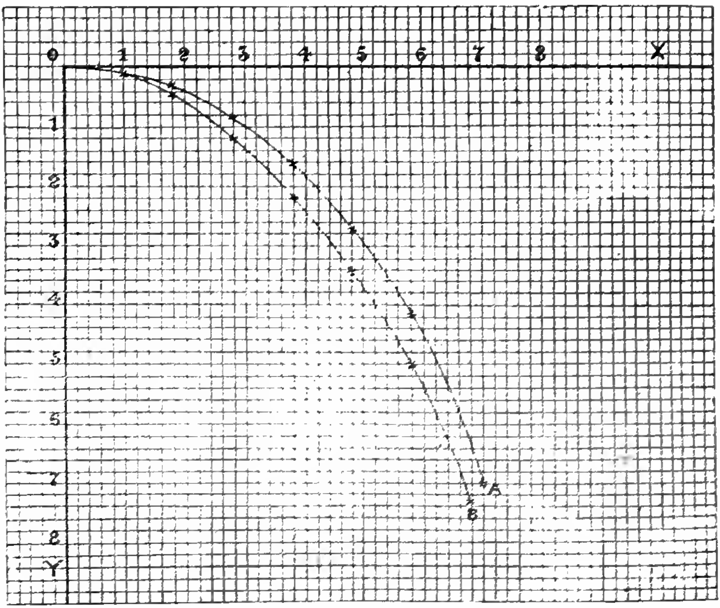


Fig 53.

* A mistake has been made in engraving the above figure; the crosses after 1 in the curves require shifting one division to the right.

Exercise.

Repeat the above experiment with different heads; see also Experiment 78 (ii.).

Experiment 77.—**Determination of the resistance to the flow of water through pipes.**

Instruments required.—A cistern of water with a long outlet pipe having lateral orifices in which are fixed glass tubes.

In Fig. 54 BC is a cistern in which the water can be

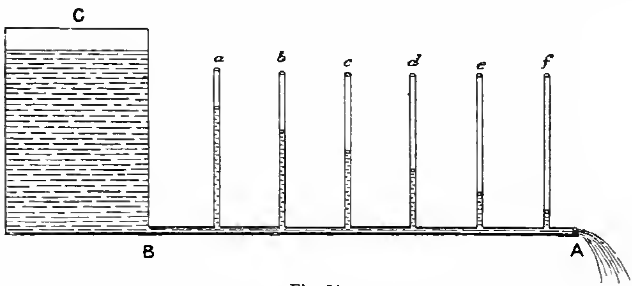


Fig. 54.

kept at a constant head by suitable arrangements, such as a ball-cock. AB is the outlet pipe having orifices terminated by glass tubes *a*, *b*, *c*, etc.

As the water flows from the cistern it will meet with resistance in the pipe AB, and rise in the glass tubes to heights proportional to the resistance in AB.

The height of the water in the glass tubes will gradually decrease from the inlet to the outlet A, where the resistance is a minimum when the tube AB is horizontal.

In making the experiment, the height of the water in

the tubes a, b, c is measured when the water is flowing uniformly through AB, and at the same time the water flowing from A in a given time is received in a beaker and weighed, and from this weight the velocity of efflux is calculated.

The results are now plotted on millimetre paper, with the distances of the glass tubes from the inlet as abscissæ, and the heights of the water in the tubes as ordinates.

Again, by varying the height of head of the water in the cistern, or the velocity of efflux by means of a tap, another curve is obtained, having the velocities of efflux as

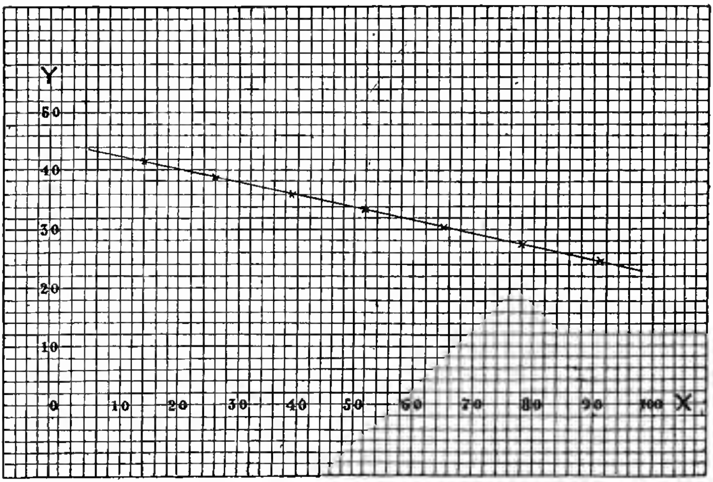


Fig. 55.

abscissæ and the heights of the water in the tube farthest from the cistern as ordinates.

Example.—The head of water being kept constant,

the following results were obtained, the distance between the tubes being 13 cms. Height of the water in the tubes in cms.

a	*b*	*c*	*d*	*e*	*f*	*g*
41·5	39·5	37·2	34·2	31·0	29·0	26·4

The results are plotted in Fig. 55.

Exercises.

1. Repeat the above experiment with AB horizontal, using different heads, and plot the result in curves.

(*a*) With distances of the tubes from the cistern as abscissæ and heights of the water in the tubes as ordinates.

(*b*) With the velocities of efflux as abscissæ and the heights of the water in the tube farthest from the cistern as ordinates.

2. With the head of water constant lower the end A of the outlet pipe, and find the angle which AB makes with the horizon, when the resistance throughout the tube is the same; that is when the heights of the water in the glass tubes are all equal to one another.

Experiment 78. — **To determine the relation between the angle of elevation, velocity, and range of a spouting jet.**

Instruments required.—A vessel of water with tap

and nipple which can be set at any angle, and a centimetre scale.

(i.) With the same arrangements for keeping the head of water constant as in Experiment 73, set the tap at various angles, and note in each case the horizontal range of the water jet. If

$$v = \text{velocity of efflux,}$$
$$\theta = \text{angle of elevation,}$$
$$\text{H} = \text{horizontal range,}$$

then *
$$\text{H} = \frac{v^2 \sin 2\theta}{g},$$

$$\therefore \frac{\text{H}}{\sin 2\theta} = \frac{v^2}{g} = \text{a constant.}$$

Example.—Prove the above law.

Enter results thus :—

θ.	Sin 2θ.	H Cms.	$\frac{\text{H}}{\sin 2\theta}$.
25	0·7660	26·2	34·3
45	1·0000	34·4	34·4
65	0·7660	27·6	36·0

Exercise.

Repeat the above experiment for every 10° from 10° to 70°

* Wormell's *Dynamics*, p. 42.

(ii.) When the jet issues horizontally a vessel similar to that in Experiment 76 may be used, having several apertures vertically above each other. Then the head being kept constant as before the *horizontal ranges* of the various jets are determined.

Now, by putting in the value of $v^2 = 2gh$ in the equation $y = gx^2/2v^2$ (see p. 183), we get $x^2 = 4hy$, where x is, in this case, the whole horizontal range of the water jet, y the height of the orifice above the horizontal plane, and h the height of the water surface above the orifice.

It is evident from the above equation that the value of x will be unaltered by interchanging the values of h and y; this simply means that the highest aperture will give the same range as the lowest, and the highest but one the same range as the lowest but one, and so on. Also, since x is a maximum when $h = y$, the greatest range is given by the central orifice, that is when the water surface is as much above the orifice as the horizontal plane is below it.

Again, it follows from the properties of the parabola that the focus of the parabolic jet will be as far below the orifice as the surface of the water is above it.

Example.—In an experiment with the jets issuing horizontally. (1) $h = 57$ cms.; $y = 7{\cdot}5$ cms. gave the range $x = 41$ cms. (2) $h = 7{\cdot}5$ cms.; $y = 57$ cms. gave $x = 40{\cdot}5$ cms. From the theory

$$x = 2\sqrt{hy} = 2\sqrt{57 \times 7{\cdot}5} = 41{\cdot}4 \text{ cms.}$$

(Experiment gives a value slightly less than this theory, as air friction affects the former.)

Exercise

Repeat the above experiment with different distances for h and y.

Experiment 79.—**To prove that, under a given head, the velocity of efflux of a liquid from an orifice is independent of its density.**

Instruments required.—A burette tube, liquids of different density, beaker, and a stop-watch.

It will have been seen (p. 178) that the expression for the velocity of efflux of a liquid is the same as that which would be acquired by a body falling freely from the upper surface of the liquid to the centre of the orifice whence the liquid escapes, the atmospheric pressure being regarded as equal at the surface and at the orifice, or $v = \sqrt{2gh}$. Inasmuch as the loss of potential energy mgh, by a certain mass m of the issuing liquid, is equal to the gain of kinetic energy $\frac{1}{2}mv^2$ by that mass,

$$\therefore\ mgh = \tfrac{1}{2}mv^2\,;\ \text{hence } v^2 = 2gh.$$

The velocity, therefore, as in falling bodies, is independent of the mass, and accordingly of the density of the liquid. A simple experiment is sufficient to prove this.

Take a piece of glass tubing about 60 cms. long and 1 or 2 cms. internal diameter, and by means of the blowpipe contract one end of the tube to an orifice about

2 mm. in diameter, *not* drawn off, to a long narrow tube, as the viscosity, etc. of the liquid then affects the flow. Make two marks on the tube some 50 cms. apart, and fix the tube vertically in a clip, with the small orifice below. Close the orifice with the finger, and fill the tube with the liquid under experiment. The finger is now removed, and whilst the liquid runs into a beaker the time when the surface of the liquid passes the marks is taken by means of a stop-watch, or by listening to the beats of an ordinary watch.

The tube is cleaned and the same operation gone through with another liquid, when it will be found that (the liquid head in each case corresponding) the time taken for equal volumes of different liquids to flow out is the same, however great their difference in density.

Example.—To show that the rate of flow from an orifice of water, mercury ($\rho = 13{\cdot}59$), and sulphate of zinc ($\rho = 1{\cdot}36$) is the same.

The stop-watch was started when the liquid surface passed the upper mark, and stopped when it passed the lower mark; this gave the time of efflux of equal volumes of each liquid from this particular tube to be 21 seconds, with a variation of less than half a second in several different experiments with each liquid.

Exercise.

Repeat the above experiment, taking the mean of three observations with each liquid.

CHAPTER X

MOLECULAR PROPERTIES OF FLUIDS:—VISCOSITY—DIFFUSION—SURFACE TENSION

In the last chapter and in Chapter V. masses of fluids have been considered, chiefly as acted upon by the force of gravity, without reference to any motion or mutual action of their molecules. In the present chapter, instead of forces acting through sensible distances on large masses of matter, we must consider the action of those forces which only *become sensible at insensible distances.* To those molecular motions and forces are due the phenomena of the capillarity of liquids and the viscosity and diffusion of fluids in general, as well as all chemical action, solution, etc.

The distance at which these forces become sensible is too small for microscopic observation or for any method of direct measurement. From observations on capillarity Quincke has deduced this distance—which may be called *the radius of molecular action*—to be about 50 $\mu\mu$ (micromillimetres); a number which lies between the upper and lower limits of molecular action determined

by Professors Reinold and Rücker by a different method of experiment.*

Within the radius of molecular action a great deviation occurs from the ordinary law of force as expressed by gravitational attraction; though the law of variation of molecular attraction is at present unknown, the magnitude of the force which comes into play is, *ceteris paribus*, vastly greater than the known forces which act upon ordinary or molar masses of matter, increasing from a small amount at the outer limit of molecular attraction to some unknown enormous amount, "which may be hundreds or thousands of kilogrammes weight before the molecules come into absolute contact." † The cohesion of bodies is due to the action of these molecular forces, and hence also elasticity, though it has been more convenient to deal with this subject in the mechanics of solid and liquid masses. In like manner the viscosity of bodies as stated on p. 171 is due to internal or molecular friction, and we shall therefore begin by a few measurements of this property.

Experiment 80.—**To determine the internal friction or viscosity of a liquid.**

Instruments required.—As described below.

Even the most limpid liquids exert a certain resistance to change of shape, depending on the rate of change; this

* See on this subject Professor Rücker's important memoir "On the Range of Molecular Forces," *Journal of the Chemical Society*, March 1888. Young in 1816 estimated this distance to be $\frac{1}{250}$ millionth of an inch.—*Young's Works*, vol. i. p. 461.

† See Lord Kelvin's *Lectures and Addresses*, vol. i. p. 11.

is due to the viscosity or internal friction of the liquid. The higher the internal friction the more nearly the liquid approaches the solid state, such as, for example, treacle, pitch, and cobbler's wax. The coefficient of internal friction, η, is the force required to slide two parallel surfaces of unit area, within the liquid, unit distance apart, through one cm., in one second of time.* The dimensions of viscosity differ from rigidity by the time unit only, the same difference that occurs between acceleration and velocity.

For many liquids the coefficient of viscosity is very small, *e.g.* water being 0·013 C.G.S. units; a rise of temperature rapidly diminishes this coefficient.

(i.) The most convenient way of determining the coefficient of viscosity is to allow a known volume of the liquid to run through a capillary tube, and take the time required. If

η = the coefficient of viscosity,
h = height of the column of liquid in cms.,
r = radius of the capillary tube in cms.,
l = length of the tube in cms.,
v = volume of liquid that flows out in unit time,
ρ = density, $g = 980$,

then †

$$\eta = \frac{h\pi r^4 \rho g}{8lv}.$$

This formula only holds true if the liquid wets the tube so that the liquid layer in contact with the tube is

* That is to say, it is the tangential force per square cm. required to maintain a constant difference of velocity of one cm. per sec. between two parallel layers of liquid moving in parallel directions and one cm. asunder.

† Jamin et Bouty, *Cours de Physique*, vol. i. part ii. p. 132.

at rest, the moving liquid therefore rubbing against itself. The whole cross-section of the liquid does not move with the same velocity, the axial portion flowing faster than the other parts. A small correction requires to be applied from the fact that the liquid leaves the tube with a certain velocity, all the work done not being spent in overcoming internal friction. As the corrections involve the square of the velocity it is best to diminish the speed by using a long fine capillary tube, which ought to be kept horizontal, the pressure height of the liquid in no case including the capillary.

The student can easily make a simple and efficient apparatus for the verification of this formula by drawing off the tube of, say, a 20 c.c. pipette to as uniform a capillary bore as possible, and then bending the capillary tube at right angles to the bulb. The capillary can afterwards be broken off and its mean diameter found by measuring the bore at its two ends, either by the Reading microscope or by means of a mercury thread (p. 46). To obtain the necessary constant hydrostatic pressure the upper part of the pipette may be connected with a cistern, when water is used, or to a reservoir of compressed air, of known pressure, for other liquids.

Example.—Find the coefficient of viscosity of water at temperature 15° C.

$$h = 300 \text{ cms.}, \quad r = \cdot 035 \text{ cms.}, \quad \rho = 1,$$

$$l = 49 \text{ cms.}, \quad v = 0\cdot 2 \text{ c.c.},$$

$$\therefore \ \eta = \frac{300 \times \pi \times 0\cdot 035^4 \times 980}{8 \times 49 \times 0\cdot 2} = 0\cdot 017.$$

As might be expected, the number obtained is somewhat larger than that given in Table XIX. The diameter of the tube used should have been less in the case of water, and the corrections referred to above would also reduce the value.*

Exercises.

(1) Find the coefficient of viscosity of alcohol, and (2) of glycerine with 10 per cent of water.

Note.—The mean velocity of a liquid in a capillary tube is proportional to the pressure (not to the square root of the pressure as in wider tubes, p. 178) and to the square of the radius. Hence as the volume discharged in a given time is jointly proportional to the velocity and to the area of the tube, it is proportional to the fourth power of the radius, not to the square as in wider tubes. This law was found by Poiseuille, and is called by his name.† By using capillary tubes of different diameter the student can verify this law, but he will find it necessary to use considerable pressure with fine tubes. This pressure may be obtained as described above, or by a column of mercury acting upon the liquid under trial through a communicating U tube, so that the mercury does not flow into the capillary.

For water, a tube about half a millimetre, or under, in diameter may be considered a capillary tube, but for

* For these corrections, see Ostwald's *Chemistry*, English ed., p. 113; see also a paper by Mr. L. R. Wilberforce, *Phil. Mag.*, May 1891.

† Poiseuille's law ceases to be obeyed when the velocity × section of the stream ÷ the viscosity of the liquid reaches a certain critical value.

liquids more viscous than water the diameter of the tube may be proportionally larger.

(ii.) Owing to experimental difficulties, such as obtaining a capillary tube of perfectly uniform bore and circular cross-section, the accurate determination of the absolute value of viscosity is very troublesome; but by taking the internal friction of water at 0° C. as unity, the relative value of other liquids can readily be found by the apparatus shown in Fig. 56, p. 198. C is a capillary tube with a bulb B, and above is an india-rubber tube with a pinch tap D, which enables the liquid to be drawn up and held in the bulb.

The tube and bulb are enclosed in a vessel of water the temperature of which is obtained by the thermometer T. The pinch tap is opened and the liquid drawn up to a mark o above the bulb; the time is taken that the liquid falls from this mark to another mark o' below, the pinch tap being closed till the experiment begins. If

t = time of flow of the given volume of liquid,

t' = time of flow of a corresponding volume of water, density = 1,

ρ = density of the liquid,

K = the relative coefficient of viscosity,

then
$$K = \frac{t}{t'}\rho.$$

Example.—Find the relative viscosity of ether of density 0·7.

$$t = 106 \text{ secs.,} \qquad t' = 276 \text{ secs.,} \qquad \rho = \cdot 7,$$

$$\therefore K = \frac{\cdot 7 \times 106}{276} = 0 \cdot 27.$$

Exercise.

Determine the relative viscosity of petroleum, olive oil, and turpentine at 15° C. and 50° C.

Note.—The effect of temperature on the viscosity of water is shown in the following series of experiments with a wide capillary tube. With a fine capillary the viscosity at 70° was found by Graham to be only one-fourth that at 0° C.

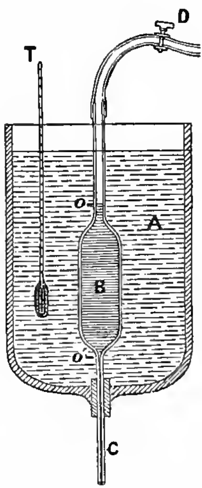


Fig. 56.

Temperature.	Time of Efflux.
1·5° C.	95 seconds.
5° C.	89 "
17° C.	74 "
28° C.	67 "
44° C.	59 "
72° C.	50 "

Plot the above numbers in a curve.

The following measurements of relative viscosity also show the effect of temperature.

Substance.	Temperature.	Time of flow.
Alcohol.	0°	275 secs.
"	60°	76 "
Turpentine.	19°	1331 "
"	53°	83 "

Experiment 81.—**Determination of the relative viscosity of liquids by a vibrating disc or vessel.**

Instruments required.—A metal disc suspended horizontally by a fine wire, and a vessel containing the liquid; also a hollow cylinder with wire suspension and long glass tubes.

(i.) If a metal disc, wetted (but not acted on chemically) by the liquids under experiment, be suspended from a firm support by a wire, and the disc made to vibrate torsionally in the liquids successively, it will be found that the periodic time of vibration will be sensibly the same in each liquid; but the amplitude will diminish like a vibrating tuning fork or pendulum, and the ratio of the amplitudes of any two successive swings in the one direction, whilst constant for the same liquid, will be different for each liquid.

Thus if in any one liquid this ratio be 0·8, this means that the second swing is only $\frac{8}{10}$ of the first, and the third only $\frac{8}{10}$ of the second, and so on. In general, the constant relation of an arc of oscillation to that next following is called the ratio of damping, and the logarithm of this ratio is called the *logarithmic decrement* of the vibrating body.

In making the experiment fix a pointer at the lower end of the wire, and allow it to move over a scale. Then observe the period of vibration of the disc in the respective liquids, and from the ratio of the observed amplitudes of the successive swings the relative viscosities of the liquids may be deduced.*

* The relative viscosity of wires (Experiment 67) may be found in a similar way by noting the *log. dec.*, wires of the same length and cross-

If t = periodic time of vibration of disc,
k = a constant proportional to the viscosity,
d = ratio of the amplitudes,
ϵ = base of the Neperian logarithms,

then
$$d = \epsilon^{-tk},*$$
i.e.
$$\log d = -tk \log \epsilon,$$
$$\therefore k = \frac{-\log_\epsilon d}{t}.$$

Example.—Find the relative viscosities of water at 15° C. and at 80° C.

Since $t = 1{\cdot}2$ secs. for both temperatures, and $d = 0{\cdot}75$ at 15° and $= 0{\cdot}9$ at 80° C.,

hence
$$k = \frac{-\log 0{\cdot}75}{1{\cdot}2} = \frac{0{\cdot}2877}{1{\cdot}2} = 0{\cdot}24 \text{ at } 15° \text{ C.},$$
and
$$k' = \frac{-\log 0{\cdot}9}{1{\cdot}2} = \frac{0{\cdot}1054}{1{\cdot}2} = 0{\cdot}088 \text{ at } 80° \text{ C.},$$
$$\therefore \frac{k'}{k} = \frac{0{\cdot}088}{0{\cdot}24} = \frac{1}{2{\cdot}7} \quad \text{(see Table XIX.)}.$$

section being successively attached to the same vibrator; or the time taken to reduce an oscillation of given magnitude by a definite amount may be noted. Thus it was found that whilst it required 8 minutes to reduce the arc of oscillation of a steel wire from 20° to 15°, it required only 6 minutes with a copper wire of the same length and thickness to reduce the arc the same amount. By this means the striking effect produced by temperature on the viscosity of iron wire (p. 157) may be verified by the student.

* For a discussion of this formula see Perry's admirable manual on *Practical Mechanics*, chap. xviii. The student should, on millimetre paper, make a series of curves showing the results of his experiments on free and damped vibrations. The manner of drawing these curves with figures is fully explained by Professor Perry, and will be referred to in the next volume, in the part on Sound.

Exercises.

1. Find the relative viscosities of water at 10° C. and 40° C. by the foregoing method.

2. Find the relative viscosities of water and a solution of zinc sulphate, density 1·2 at 15° C.

(ii.) The viscosity of liquids may also be compared by suspending a vessel containing the liquid by a fine wire, with attached pointer moving over a graduated circle. The wire is turned through a given angle and released, so that a torsional vibration is set up, and the logarithmic decrement found as above, or the *time* taken to reduce the vibration through any definite angle is accurately determined, the same vessel being employed for the liquids compared.

An unboiled and a hard-boiled egg thus compared form an instructive and striking experiment, the former coming to rest immediately. The reason for this is obvious. A boiled egg acts as a solid body, but an unboiled egg behaves like a vessel enclosing a very viscous liquid. The inertia of the liquid opposes the changing motion of the oscillating vessel, and the greater the viscosity the greater the frictional resistance. The amplitude of successive oscillations being reduced in a geometric ratio, the logarithmic decrement measures the relative viscosity.

Employing a hollow cylinder with plane ends suspended axially by a fine wire, or by a bifilar

suspension, accurate absolute measurements of viscosity have lately been made by O. E. Meyer and others,* and some interesting relations between the coefficient of viscosity of solutions and their chemical properties have been found. But as yet no general relation between the internal friction of liquids and their chemical constitution has been arrived at, though much labour has been spent on this subject.

Exercise.

Compare in this way the viscosity of water and engine oil, also boiled and unboiled eggs as above.

(iii.) Another method of exhibiting the difference in the viscosity of liquids is by filling a long glass tube, closed at one end, with the liquid under experiment, and note the time taken for a bubble of air, just large enough to touch the sides of the tube, to rise to the top of the tube. In this way various liquids may be compared with water. It is best to cork the tube, leaving a small air space over the liquid, then, stop-watch in hand, invert the tube. The ascent of bubbles in a tube of liquid presents many interesting points, such as a remarkable periodic fluctuation in the speed of ascent as the length of the bubble of air is increased; these phenomena have been investigated and ingeniously explained by Dr. Trouton of Trinity College, Dublin.

* See O. E. Meyer and Mützel in *Wiedemann's Annalen*, vol. xliii., where the student will find a complete expression for the investigation of this subject.

Exercises.

1. Compare, by the rate of ascent of a bubble of air in a tube of the liquid, the viscosity of water containing various proportions of glycerine, and plot your results on millimetre paper.

2. Keeping the solution constant, vary the size of the bubble and note the speed of ascent: plot your results in a curve.

Experiment 82.—**Determination of the relative viscosity of gases by their rate of transpiration.**

Instruments required.—Similar to that in the succeeding experiment, the pinhole being replaced by a long capillary tube, and the pressure and temperature of the gas alike in each experiment.

When a gas flows through a capillary tube the process is called *transpiration*, and, like the corresponding phenomenon in liquids, the rate of flow is independent of the nature of the tube; a film of gas adheres to the sides of the tube, and the gas flows in a stream along the axis, being impeded by its own internal friction or viscosity. The rate of flow is inversely proportional to the length of the tube, and to a constant η, called the coefficient of viscosity, which is peculiar to each gas. Graham has found the singular result that "any cause which promotes the density of a gas increases its velocity of transpiration, *i.e.* decreases its viscosity, whether the increased density be due to a lower temperature, to compression, or to the addition of an element in combination,—*e.g.* the viscosity of CO_2 is less than O_2." Hence whilst rise of temperature

decreases the viscosity of liquids it *increases* that of gases. The rate of transpiration is very different from that of effusion (Appendix, § 16).

As in the case of liquids the determination of the absolute coefficient of viscosity involves difficulties, which do not exist in a relative estimate; this latter may easily be made by an arrangement such as that described in the next experiment, a length of fine thermometer tube furnishing the transpiration tube to be used in each experiment. When the same volume of different gases are transpired through the same tube under the same pressure, we have $\eta/\eta' = t/t'$, where t and t' are the transpiration times.

Example.—Find the relative viscosity from the transpiration times of hydrogen and oxygen.

For H_2 the time was found to be 51 secs.,
„ O_2 „ „ 115 „

or calling O = 1, as 1 : 0·44 (see Table XX.).

Exercise.

Find the relative viscosity of air, coal gas, and hydrogen.

Note.—The viscosity of gases may also be determined by the vibration of discs in the gas; this method was employed by Maxwell.*

The viscosity of gases compared with liquids is much greater than their relative densities would suggest. Thus whilst water is 770 times denser than air, its viscosity is

* The student should read Maxwell's *Heat*, new ed., chap. xxi.

only 100 times greater. Barus* has measured the change of viscosity which occurs in passing from the gaseous to the liquid state, thus he finds η for ether vapour at 0° C. $= 6{\cdot}8 \times 10^{-5}$, for liquid ether at 30° C. $= 9 \times 10^{-4}$. He also finds η for marine glue $= 2 \times 10^{8}$, for steel $= 6 \times 10^{17}$; note the enormous range from hydrogen to steel.

Experiment 83—**To determine the velocity of effusion of a gas.**

Instruments required.—As described below.

When a gas flows through a small aperture (not more than 0·013 mm. diameter) in a thin plate, say of metal or glass, the process is called *effusion*, and the velocity of efflux obeys the same law as liquids, viz., $V = \sqrt{2gh}$. Air at 0° and 76 cm. pressure, rushing through such an aperture into a vacuum, will therefore move at the rate of $\sqrt{2 \times 981 \times 790000}$, $= 39380$ cms. per second, the value of h in this case being the height of a homogeneous atmosphere, or 790,000 cms., nearly 5 miles. With different gases the velocities will be inversely proportional to the square root of their densities, *e.g.* at the same temperature and pressure the rate of effusion of oxygen to hydrogen will therefore be as 1 : 4; accordingly the densities of two gases are inversely as the squares of their velocities of effusion. By this means the relative densities of gases and vapours may be determined as follows: Into a tall vessel (A) containing mercury (Fig. 57) a glass tube (B) is plunged.† To the upper end of

* *Phil. Mag.*, April 1890.

† A lamp chimney closed at the upper end with a piece of tinfoil, in which a fine hole has been pricked, and the whole immersed in a vessel of

the tube is cemented a piece of platinum foil C, in which a fine hole has been pricked by a needle point. Closing this with the finger, the tube is filled with mercury, and the gas whose effusion rate is to be measured is allowed to displace the mercury. A bit of glass rod (F), drawn out with a long stem, is introduced, and floats on the mercury in the tube B. On the stem are two marks; the upper mark is brought level with the mercury in the outer vessel by allowing a little of the enclosed gas to escape from the pin-hole; now removing the finger from C the time is noted which elapses until the lower mark is level with the mercury. The tube B must be rigidly held in a clip during the experiment.

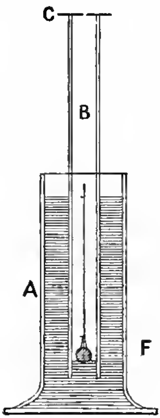


Fig. 57.

This method can be used for testing the density of different specimens of coal gas, and may be varied in this case by allowing the gas to stream from a constant pressure gasholder through a small gas-meter connected with the tube B by flexible tubing. The quantity of gas that escapes in a given time being thus measured.

Example. Find the relative effusion rate of hydrogen and oxygen (see Table XXI. and Appendix, § 16).

Time for hydrogen = 20 secs.,
„ „ oxygen = 77 „

water or brine, serves very well. The finest needle must be used to prick the hole, which may be made smaller by a gentle tap with a hammer, the foil lying on a smooth surface. If the hole is too large the motion of the gas is tumultuous, and the law of effusion is not obeyed.

The ratio of the squares of these numbers should express the relative densities of the two gases, *i.e.* $\frac{20^2}{77^2} = \frac{1}{14 \cdot 8}$ would be, in a more accurate experiment, 1/16.

Exercise.

Determine the relative densities, from the rates of effusion, of hydrogen, coal gas, and CO_2.

Experiment 84.—**To determine the law of the diffusion of gases.**

Instruments required.—A diffusion tube or bulb, closed at one end by a porous septum as described below, a balance, gramme weights, beaker of water and bell jars or small holders containing a few different gases.

At temperatures above the absolute zero, the molecules of all bodies are in rapid motion; whilst in solids this motion is restrained within very narrow limits, in liquids and gases it is not so. Hence any two or more gases, or any miscible liquids put in contact rapidly intermingle. This molecular intermingling is due to the transference of individual particles from place to place throughout the mass, and is called *diffusion.* This process is far more rapid in gases than in liquids, owing to the greater molecular freedom in the former, but it is a slow process compared with the intermingling of masses of liquid or gas by convection, which is a molar and not a molecular motion. The rate of diffusion, as of effusion, of any gas is inversely proportional to the square root of its density.*

* See Daniell's *Physics*, p. 233, and Maxwell's *Heat*, chap. xxii.

To illustrate diffusion the following experiments may be made. (1) Connect two soda-water bottles with a long narrow glass tube and well-fitting corks, fill one of the bottles with hydrogen and the other with oxygen gas, and place the bottles (vertically connected together) aside, the H_2 being uppermost. The next day test the gas in each bottle by a lighted taper; an explosive mixture will be found in each, showing that the hydrogen has intermingled with the lower heavier gas. (2) Take a small cylindrical porous jar, such as is used for Daniell's batteries, and a wide-mouthed bottle half full of water; close both by a cork, through which pass a glass tube, so that the porous pot is supported vertically over the bottle by the glass tubing, which may be a foot or so long. Fix a second piece of glass tubing in the cork of the bottle, so that one end dips below the surface of the water, the upper end being drawn off to a jet. Fill a bell jar by displacement with H_2 or coal gas and hold it over the porous pot; immediately diffusion begins, the H_2 passing more rapidly in than the air out, hence a pressure is produced inside the porous pot in excess of the atmosphere; this causes the water in the bottle to be driven with considerable force up the second tube, so that a fountain plays from the jet.

To verify the law of diffusion it is best to allow the gas to diffuse through a porous septum, such as plaster of Paris, compressed graphite, or better still, "biscuit ware," *i.e.* any fine kind of unglazed earthenware, like the porous pot in the last experiment. For this purpose a *diffusion tube* is made as follows. A glass tube, some 20 or 30

cms. long and some 2 cms. diameter, graduated like a burette (p. 47), is closed at one end by a disc cut from a broken porous pot, the disc being cemented in with sealing-wax a little below the end of the tube and a cork tightly fitted above it. The tube is now filled with the gas under trial as follows: pass one limb of a bent glass tube (like an inverted siphon) up the diffusion tube till it touches the disc, the air within can thus escape when the tube is lowered in a vessel of water, and in the same way the gas under trial can be made to enter and displace the water. Care must be taken not to wet the disc. When the tube is full of gas remove the cork and allow diffusion to take place; with gases lighter than air water will rapidly enter the tube, and after half an hour the volume of the remaining gas is read off, the tube being depressed until the level of the water within and without is the same.* With gases heavier than air the tube should only be half filled, and the increased volume of gas read off in the same way. The temperature should remain constant, and a loose cone of damp paper should be placed over the disc, so that the humidity of the entering gas may be the same as that of the escaping.

A more accurate experiment may be made with a *diffusion bulb* (Fig. 58). The bulb B may be some 5 or 6 cms. diameter, having a neck above, say 3 cms. diameter, and for convenience the neck below may be narrower. As before, close the upper neck or opening by a disc of biscuit-ware A, cemented a little below the top, so that when filling the bulb it can be stopped by a rubber cork;

* This should be the case throughout the experiment, see next page.

fill the whole with gas as before by means of a bent glass tube, and allow diffusion to occur under constant pressure and temperature. The weight and capacity of the bulb being known by weighing it empty and full of water, the weight of water which enters or leaves the bulb, and hence the "diffusion volume," is found by closing the lower end with a cork and weighing the bulb and water in it. To preserve the hydrostatic pressure constant during diffusion suspend the bulb by a thread t from one arm of a balance, and having counterpoised it allow the lower end to dip in a beaker of water, as shown in Fig. 58. As water enters or leaves the bulb the movement of the balance preserves a constant water-level within and without.

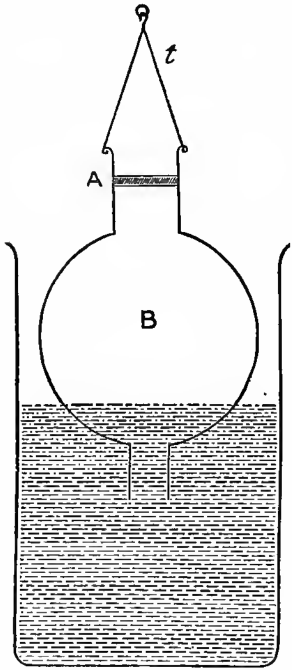


Fig. 58.

If V_0 be the original volume of the gas, and V_1 the volume remaining after diffusion (*i.e.* the volume of the return air), then the *diffusion volume* of the gas is V_0/V_1, and the density of the gas compared with air is $\rho = (V_1/V_0)^2$.

Example.—To find the diffusion volume and density of hydrogen compared with air.

A bulb diffusion tube was filled with hydrogen in the manner described above. The weight of the bulb containing air $= 41·4$ grammes; when full of water $= 147·9$ grammes. The capacity of the bulb was therefore $147·9 - 41·4 = 106·5$ c.c. approximately. After one hour diffusion ceased, and the bulb was weighed again, when 78·7 c.c. of water were found to have entered the bulb. Hence $106·5 - 78·7 = 27·8$ c.c. of air had replaced 106·5 c.c. of hydrogen.

$\therefore$ $106·5/27·8 = 3·83 =$ comparative rate of diffusion, or *diffusion volume* of H_2, and $(27·8/106·5)^2 = ·076 =$ density of hydrogen.

From Table XXI. the density of H_2 compared with air is seen to be $·0693 = \frac{1}{14·43}$; the calculated diffusion volume would therefore be 3·795, the square of which is 14·4.* (See Appendix, § 16.)

Exercise.

Find the rate of diffusion, and the density compared with air, of H_2, CO_2, and coal gas.

Experiment 85.—**On the diffusion of liquids.**

Instruments required.—Solutions, glass jars, and sp. gr. beads.

Diffusion takes place between all gases and between those liquids which mix with each other. When, for

* See Graham's *Researches*, collected and edited by Dr. Angus Smith, p. 64. To these famous investigations our present knowledge of the diffusion of gases and liquids is chiefly due. The number found (3·83) happens to be the same as the mean of Graham's observations.

example, a layer of pure water rests over brine, the dense salt solution immediately begins to rise into the water and continues to diffuse until the salt is uniformly distributed throughout the whole mass of water. Different soluble bodies have different rates of diffusion. Their diffusibility may be compared, and the laws of liquid diffusion studied as follows: take a small wide-mouthed bottle or beaker with a ground top, fill this "diffusion jar" with the solution to be tested, cover it with a ground glass lid, and lower it to the bottom of a large beaker of distilled water. When the water has come to rest gently slide off the lid by means of a wire, note the time, aud leave the arrangement undisturbed and in a uniform temperature, which note. After an hour carefully draw off by a pipette a measured amount, say 20 c.c. of the water from two different and measured distances above the top of the diffusion jar. Evaporate each sample to dryness in an evaporating basin and weigh the residue; this corresponds to the amount of the substance diffused in an hour. Take other samples at other longer intervals of time from corresponding layers above the diffusion jar and test these; and repeat, if possible, when the temperature of the air differs considerably.

In this way the quantity of the substance diffused will be found to depend on (1) the length of time; (2) the strength of the solution in the diffusion jar; (3) the temperature—being greater with a high temperature—and (4) the coefficient of diffusibility peculiar to each substance. The diffusivity of one substance in another may be defined as "the number of units of the substance

which pass in unit of time through unit of surface, across which the gradient of concentration is unit of substance per unit of volume per unit of length."* Making the unit of time a day, instead of a second, to avoid inconveniently small numbers, the following numbers have been deduced from Graham's experiments on diffusion.

	Temp. C.	Gramme per day.	Ratio of Times.
Hydrochloric acid . .	5°	1·74	1
Common salt . . .	5°	0·76	2·33
" . . .	10°	0·91	...
Sugar	9°	0·31	7
Albumen . . .	13°	0·06	49
Caramel . .	10°	0·05	98

The last column gives the ratio of the times required for diffusion into water of equal amounts of the substances named. The third column means that if the vessel of water were divided into horizontal layers 1 centim. apart, and each layer had 1 gramme per c.c. more of the substance named than the stratum above, the upward diffusion of the substance would be the number of grammes given through each sq. cm. *per day*.

The process of liquid diffusion may also be investigated by using small sp. gr. beads of differing densities and observing from day to day the position where each floats. The beads can be made with paraffin wax having a little tail of copper wire stuck in; they require careful adjustment—first by nipping off bits of the wire,

* Tait's *Properties of Matter*, p. 257.

and finally by removing shreds of paraffin. As air-bubbles are apt to collect on them it is desirable that the liquids under diffusion should first be boiled, and the whole covered with a layer of oil to exclude air; a smart tap will, however, generally remove the air-bubbles without disturbing the liquids. Hydrometer jars or glass tubes closed at the lower end may in this case be used for diffusion. The jar or tube is first partly filled with water, and then the solution, if denser than water, is slowly poured down a funnel-tube reaching to the bottom of the jar; the funnel-tube need not be removed, but if it is, care must be taken to close the upper end before withdrawing it. When highly coloured salts, such as bichromate of potash or sulphate of copper, are employed, the process of diffusion can be watched day by day, the upward progress of the dense layer forming a continuous gradation of tint, analogous to the fall of temperature in a metal bar heated at one end—the law of diffusion of heat being the same as that of the diffusion of matter. It will be found that the process of diffusion is very slow, going on for weeks and even years in a long glass tube. The rapid intermixture of liquids produced by stirring arises from the enormously increased area of diffusion and the greatly thinner layers across which diffusion has to take place.

Various optical methods of estimating the rate of diffusion have been tried, such as enclosing the liquid in a hollow prism and taking the refractive index at the vertically placed refracting angle. In coloured solutions the absorption of light by the liquid, placed in a

vessel with parallel glass sides, might be tried by the student.

Exercise.

• Find the rate of diffusion into water of saturated solutions of (1) common salt, (2) sulphate of zinc, (3) sulphate of copper, and also (4) of alcohol.

Experiment 86.—**On the osmose of liquids.**

Instruments required.—As described below.

When a semi-permeable diaphragm, such as parchment paper or bladder or a porous earthenware partition, separates two miscible liquids, the process of free diffusion is modified. Thus if a bladder be partly filled with alcohol, or a piece of bladder tied over the end of a glass tube or lamp chimney and alcohol poured in, and the bladder then immersed in a vessel of water, the water enters and distends the bladder or raises the level of the liquid in the tube; the reverse effect occurring if the liquids be interchanged.

If, instead of bladder, a piece of thin caoutchouc or a toy rubber balloon be used, the alcohol and not the water passes through the rubber. The reason is that whilst water moistens or is soluble in bladder, alcohol hardens, and is not soluble in, bladder; whilst the reverse is true of caoutchouc. The solution of the liquid in the septum creates a pressure—*osmotic pressure*—which drives the liquid into the solvent. The following experiment further illustrates this action :—

Tie a piece of bladder over the mouth of a glass

funnel; to the shank attach say two feet of glass tubing by means of a rubber connection. Pour some thin treacle, or syrup, or a saturated solution of sulphate of copper into the funnel before attaching the tube; by means of a clip support the funnel within a beaker; pour water in the beaker till it rises above the mouth of the funnel, and mark the level of the treacle which should stand in the stem of the funnel. In the course of a few hours the liquid will be found to have risen in the tube attached to the funnel, and after a day or two may even reach the top of the tube and overflow. Simultaneously some of the solution has diffused out into the beaker of water, though at a slower rate.

The difference in the rate of diffusion between crystalloids and colloids * enables a separation to be made between a mixed solution of these bodies. This process is termed *dialysis*. A dialyser can be formed by floating a small toy tambourine, or suspending an inverted funnel with a piece of bladder or parchment paper tied over its mouth, in a beaker of pure water. Into the hoop or funnel pour a solution of gum arabic mixed with bichromate of potash or common salt; after a few days the salt, together with saline impurities in the gum, will have entirely diffused through leaving a pure solution of gum on the dialyser. The following results were obtained by Graham with 100 c.c. of different solutions, each containing 10 grammes of the substances named.

* Graham's *Researches*, p. 552; or *Phil. Trans.*, 1861.

Dialysis through parchment paper in 24 hours at 10° to 15° C.

Ten per cent Solution.	Diffusate in Grammes.	Relative Diffusate.
Chloride of sodium . .	7·50	100
Glycerine	3·30	44
Starch sugar	2·10	27
Cane sugar	1·61	21
Milk sugar . .	1·39	18
Gum arabic	0·03	0·4

Exercise.

Make two solutions, one containing sugar and the other an equal quantity of common salt; after 24 hours compare their rates of diffusion through two dialysers, by evaporating in a water bath the liquid on each dialyser and weighing the residues.

Note.—A difference of electromotive force on the two sides of the porous septum also causes osmose, the liquid passing in the direction of the electric current; in this way with liquids of high electrical resistance, like water and alcohol, a rapid molecular transport is produced.

Surface Tension

In the interior of a mass of liquid the molecular forces acting on any particle of the liquid balance one another, but the particles forming the external surface of the liquid will not remain in the same kind of equilibrium. These particles may be considered as forming

"a thin coating of the liquid surrounding a substance which resists only in a direction perpendicular to its surface; . . . this coating must exert a force on the points in contact with it precisely similar to that of a flexible surface, which is everywhere stretched by an equal force; and from this simple principle we may derive all the effects which have been denominated capillary attraction."*

Hence the surface of a liquid behaves as if under the influence of a contractile force, which tends to reduce it to the smallest possible area, and exerts a pressure on the interior which is greater when the surface is convex and less when it is concave than when it is plane. The surface of every detached portion of a liquid must, in fact, everywhere have such a curvature as to be able to withstand the hydrostatic pressure which acts against it.

If the surface of water be supposed to be divided into two parts by an ideal boundary, each of the two surfaces tends to enlarge the other and to contract itself. The force exerted by each is at right angles to the boundary and equal to nearly 0·08 grammes per lineal centimetre. Over the free surface of the liquid this force is uniform, and its amount depends on the nature and temperature of the liquid. The magnitude of this force is however modified by the contact of the liquid surface with the surface of another liquid or with the surface of a solid.

All capillary phenomena can be explained by assuming

* Young's *Lectures* (delivered in 1807), p. 475. To the genius of the learned and famous Dr. Thomas Young we largely owe the development and application of the surface tension of liquids. For the modern treatment of the subject of capillarity the article in the *Ency. Brit.* should be read, or chap. xii. in Tait's *Properties of Matter.*

the existence of this force and the ordinary hydrostatic laws. As capillarity occurs *in vacuo* a definite molecular pressure appears therefore to be exerted by the body of a liquid. This internal pressure, R, may be regarded as a measure of, and due to, the cohesive forces within a liquid, the range of which is limited to the "radius of molecular action." The value of R is probably enormous, Young estimated it at 20,000 atmospheres.

In addition to the *elastic* skin which surface tension appears to confer upon the free surface of a liquid, the superficial film may possess a certain degree of *toughness* which renders it hard to displace or break; this was supposed to arise from the fact that the surface of most liquids had a greater viscosity than the interior mass. The frothing of liquids and the formation of a soap bubble were attributed to this *surface viscosity.* Certain substances, such as albumen and saponine (infusion of horse-chestnuts) do appear to confer on water a kind of *surface rigidity*; the surface film having more the properties of a solid than a liquid. A bubble blown with saponine wrinkles up when the air within is withdrawn, behaving therefore quite differently from a soap bubble, the tension of which is the same in all directions of its surface. A sewing needle can easily be floated on a solution of saponine, and if magnetised the superficial rigidity of the solution will prevent it obeying the directive force of the earth. The apparent "superficial viscosity" of water has recently been shown by Lord Rayleigh * to be due to the almost invariable contamination

* *Proc. Royal Society*, March and June 1890 ; see Appendix, § 18.

of its surface by a kind of greasy film, which can only be removed by special precautions (see p. 236, and Appendix, § 17); when these are taken the water is then found to be devoid of "superficial viscosity." The addition of soap to water greatly diminishes its surface tension and increases its "surface viscosity." Both these effects are doubtless due to the formation of a film or pellicle on the surface, which cannot be skimmed off, for it is renewed from the interior as fast as it is removed. But as the formation of this pellicle takes a certain, though very small interval of time, when soapy water is tested before its surface is $\frac{1}{100}$ of a second old, it will be found to have the same tension as pure water. In the feebler tension of this outer renewable pellicle is to be found the probable explanation of why soap and some other substances confer on water the property of enabling a fairly durable bubble or stable extended film to be formed.*

Liquid veins also afford beautiful illustrations of superficial tension; their constitution and sensitiveness to sound and electricity will be studied experimentally in the next volume.

Experiment 87.—**On the direct measurement of surface tension.**

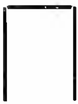
Instruments required.—A balance and weights, wire bent as described, and a beaker of water and soap solution.

(i.) The following experiment directly illustrates the existence of surface tension and affords a measurement of

* For a further explanation see Maxwell's *Heat*, new ed., p. 298, and Appendix, § 18.

its amount, well adapted for soap films; other methods of determination will be subsequently described under the capillary constants of liquids.

Bend a piece of wire into a rectangular fork, thus,



the width being from 2 to 3 cms. and the length, say, 5 or 6 cms. Suspend it by a thread (attached to the cross piece) from one arm of a balance, allowing the legs to dip into a beaker of pure clean water. Counterpoise the wire so that the cross piece is about 2 cms. above the surface of the water when the balance is in equilibrium. Next, depress the balance beam so that the wire dips below the water surface—the counterpoise will now be found unable to restore the equilibrium; add weights gently. A film of water will be drawn up in the rectangle of the fork until equilibrium is restored. Find the greatest stretching weight the film will bear without breaking; this can be done, after one or two trials, by altering the level of the water in the beaker and readjusting the counterpoise.

Note that as the film thins the stretching force required is just the same (allowing for the very slight loss of weight it sustains by evaporation, etc.) till the breaking point is reached. A thin sheet of india-rubber would, of course, behave differently, requiring less force to stretch it than a thicker sheet; and the tension of such a sheet would grow less as it shrinks, whereas the tension of the liquid film remains the same and has therefore a certain definite value.

Next, try in the same way a soap solution (Appendix, § 20). A much larger and tougher film can be obtained;

but though the film is bigger, a less stretching force is required, that is, the contractile force of the film pulling the fork back into the solution is less. As the film has two surfaces the tension per cm. is double that of the free surface of the liquid in the beaker. The stretching force F is distributed over the breadth AB of the film; hence if T be the surface tension *per unit of length* $F = 2T \times AB$,

$\therefore$ $2T = F/AB$, where F is the stretching force in dynes.

Example.—Find the surface tension of pure water and of soapy water.

A fork 3 cms. broad was suspended in a beaker of pure water and counterpoised. At this temperature, 19° C., the maximum pull of the water film was 0·37 gramme. The same fork being used in the glycerine-soap solution the maximum pull of the film was 0·18 gramme. (The weight of the film itself, which was 5 times the area of the water film, should be deducted.)

Hence for water at 19° C.

$$T = \frac{0·37 \times 981}{2 \times 3} = 60·4 \text{ dynes per lineal cm.,}$$

and for soap solution, neglecting the weight of the film,

$$T' = \frac{0·18 \times 981}{2 \times 3} = 29·4 \text{ dynes per lineal cm.}$$

The value of T for water is here rather low, doubtless owing to some impurity on the surface; but it will nevertheless be seen that the soap has reduced the surface tension of the liquid one-half.

Exercise.

Repeat the above experiment.

(ii.) Another method of measuring the surface tension of a soap film is as follows: *

Make two wire rings, one about 5 cms. diameter, the other a little larger; support both by three wire legs, the legs of the larger ring about 5 cms. long and the smaller about 2 cms. long; fix to the legs of the smaller tripod, and parallel to the ring, a paper disc or tray. Now wet the rings with the soap solution and form a film on the larger ring; raise the lower ring till it is in contact with the film and concentric with the larger ring; let it go, and break the film within the smaller ring; absorb by filter paper any drops of liquid, and gently load the paper tray with sand. The annular film is drawn down into the shape of a catenoid. Continue loading with sand, preserving the lower ring horizontal, till the tangent to the base of the curved film is little removed from the vertical. When it is practically vertical, the surface tension pulling the tray up acts wholly vertically and is equal to the weight w (which is then at its maximum value), pulling the film vertically down; hence

$$2\mathrm{T} = \frac{wg}{2\pi r},$$

T being in dynes per lineal cm. and acting along the circumference of the ring whose mean radius is r in cms. (*i.e.* one-half the sum of the radii of the external and

* Van der Mensbrugghe in *Phil. Mag.*, April 1867, p. 280.

internal rings). When the legs of the lower ring rest on the table, break the film and weigh the whole load.

Example.—Find the surface tension of a soap film by Van der Mensbrugghe's method.

The mean of five experiments gave a load of 1·01 gramme, $2\pi r = 16{\cdot}75$.

$$\therefore \quad 2\text{T} = \frac{1{\cdot}01 \times 981}{16{\cdot}75} = 59{\cdot}2,$$

and $\text{T} = 29{\cdot}6$ for a glycerine-soap solution.

Experiment 88.—**Various illustrations of surface tension.**

Instruments required.—As described below.

The following additional experiments, which should be repeated by the student, afford instructive illustrations of the action of surface tension.

(1) Make a wire ring some 5 cms. in diameter, leaving a straight piece of wire for a handle; dip the ring in a soap solution, and by removing it obtain a film, covering the ring. Throw a thread wetted in the soap solution across the film; it will move about freely, being drawn equally on all sides; now break the film on one side of the thread,—instantly the thread is pulled across the ring by the contractile force of the remaining portion of the film (to make the soap solution, see p. 261).

(2) Tie the ends of a short length of thread, place the loop thus formed, wetted, on the film, and break the film within the loop; the uniform tension of the film outside instantly pulls the loop into a perfect circle.

(3) Blow a soap bubble from the end of a wide glass tube, close the free end of the tube with the finger; on removing the finger the bubble contracts and can be made to blow out a candle.

(4) Blow a bubble at the end of a glass T-piece, to the other horizontal limb of which is attached a small U-tube containing water, to indicate the pressure; note that the greatest pressure (about $\frac{1}{4}$ inch) is when the bubble is small—owing to the greater curvature of a small bubble. From this pressure and the size of the bubble the surface tension of the soap film can be deduced.

(5) Make a solution of zinc sulphate of the same density as bisulphide of carbon, and with a pipette allow a drop of bisulphide of carbon, coloured by iodine, to enter the solution; note the spherical form of the drop. A liquid sphere of olive oil of considerable size may be similarly obtained in a mixture of alcohol and water. The drop being freed from the influence of gravitation the spherical shape is the result of the action of the molecular forces alone. By increasing the density of the solution, a large drop may be floated on the surface; note its flattened form, as gravitation now plays a part.

(6) Spread some coloured water on a plate, drop into it a little alcohol, or hold a red-hot rod, or a rod dipped in ether, over the water; immediately a bare spot is left beneath the rod. The stronger surface tension of the pure water draws the liquid away on all sides from that portion which has had the contractile force of the film weakened by the solution of alcohol or ether, or by a rise of temperature.

(7) On the clean surface of pure water contained in a wide beaker, place a drop of creosote; the globule floats, and a partial solution in the water begins. Instantly the surface tension is lowered and the globule is pulled to pieces and drawn rapidly about by the higher tension of the water around; hence also ensues a rapid vibratory and singularly life-like motion of the drop, particles being shot out in radial lines like the scattering of spores. Touch the water with a rod dipped in turpentine; immediately the whole is struck lifeless, as a turpentine film of much lower surface tension than water flashes over the surface. Each essential oil has its own "cohesion figure." These have been investigated aud drawn by Mr. Charles Tomlinson,* and afford a method of discriminating and testing the purity of essential oils.

(8) Fragments of camphor floating on clean water exhibit similar lively motions, owing to the lowering of the tension of the water surface at those points where the camphor dissolves most freely. A trace of oil or turpentine on the water stops the movement. The importance and difficulty of obtaining a water surface free from contamination is referred to in the Note on p. 236.†

Before making more exact measurements of surface tension it is necessary to investigate the law of the ascent or depression of liquids in capillary tubes.

* *Phil. Mag.*, June 1867.

† Other striking and beautiful illustrations of surface tension are contained in Professor Boys' charming book on *Soap Bubbles*, and in a paper by Lord Rayleigh in the *Phil. Mag.*, April 1892.

Experiment 89.—**To prove that the height of ascent of water in capillary tubes is inversely as the radius.**

Instruments required.—A vessel of water, a millimetre scale or cathetometer, and several capillary tubes.

(i.) Take three or four capillary tubes of different sizes and clean them internally by dipping them into strong sulphuric acid several times, wash with distilled water, then alcohol, and finally blow air through them, heating them at the same time in a Bunsen flame.

It is necessary that the tubes should be of uniform bore and circular cross-section. Ordinary thermometer tubes answer very well, but the student had better prepare his own tubes by softening a piece of quill-glass tubing in a blow-pipe flame; then remove the tubing from the flame and draw it out quickly and steadily to a long length. Selecting about six inches of the central part of the capillary, test it for uniformity of bore, and measure its diameter as described in Experiment 17.

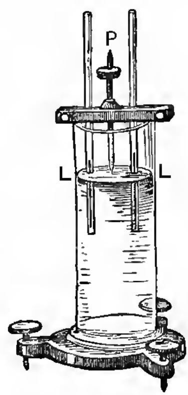


Fig. 59.

Place the tubes *vertically* in the vessel of water (Fig. 59); raise the liquid in the tubes to a height above that which it will ultimately take (by first lowering the tubes and then raising them); this ensures that the inside of the tubes are thoroughly wet.

Now measure by means of the millimetre scale or the cathetometer the heights of the liquid in the tubes above

that in the vessel, the latter can be given by a fine point P, screwed down to the level of the liquid L: it will be found that the heights are inversely as the radii of the tubes. Then the height h, multiplied by the radius r, should be a constant * (see Appendix, § 19).

Example.—Prove the law of ascent, using three capillary tubes.

Enter results thus:—

Number of Tube.	h Cms.	r Cms.	hr.
1	4·2	·0317	·1332
2	3·1	·0435	·1349
3	2·7	·0489	·1320

The water used in the foregoing experiment was slightly coloured. When very clean water is used, and a perfectly clean vessel and tube, the value of hr is higher and will be found to be nearly 0·15—a constant higher than that given by any other liquid. The experiment should be repeated in water at different temperatures; the capillary elevation will be found to diminish as the temperature rises; the surface tension of water, on which capillarity depends, diminishing rather more than $\frac{1}{500}$ of its value for each degree centigrade above 0°. Without special precautions it

* For exact measurement it is necessary to take into account the height of the liquid meniscus in the tube. By taking the reading a little above the bottom of the meniscus (say one-third of the radius of the tube), this error vanishes compared with the greater errors due to want of perfect cleanliness of tube and liquid surface.

is difficult to keep the liquid in the tubes at temperatures above that of the air; but by using thick glass tubes and allowing them to attain a temperature somewhat higher than the water used, comparative experiments may be made if quickly performed.

(ii.) Instead of using *tubes*, the law of elevation (sometimes called Jurin's law) may be determined by using glass plates. When two parallel glass plates, a small distance apart, are immersed in water, the liquid rises to a height which is inversely proportional to their distance asunder, hence the height of ascent is *one half* what it would be for a tube whose diameter corresponds to the distance between the plates.* The best way of keeping the plates a measured distance apart is to cut two short lengths of wire, say 1 mm. diameter, place the wire between the plates near the edges which are vertical when in the liquid, and bind the whole together by a couple of elastic bands. Wires of other thicknesses enable the distance to be varied.

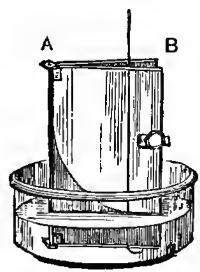


Fig. 60.

Glass plates inclined at a small angle may also be used as follows:—Take a shallow vessel of coloured water and place two plates of glass vertically in the vessel, as in Fig. 60. The two plates touch along one edge as at A, and are a little apart at the opposite edges B. When they are placed in position the water will rise

* Deschanel's *Physics*, edited by Everett, p. 128.

between them in the form of an equilateral hyperbola. Now measure the ordinates of a few points of the curve, and show that their lengths are inversely proportional to the distances between the plates at these points, that is the same law as in parallel plates.

In liquids which do not wet glass, like mercury, a corresponding *depression* of the liquid is produced by capillary tubes and plates of glass. Jurin's law will here be found to be only approximately true owing to the disturbing influence of the unwetted walls of the tube. The capillary depression of mercury in glass tubes is given in Table X.

Exercise.

Repeat the above experiments with capillary tubes and parallel and inclined plates of glass.

Experiment 90.—**Determination of the capillary constants of liquids.**

Instruments required.—As in the last experiment, with specimens of different liquids.

The most convenient and accurate way of determining the surface tension of a liquid which wets glass, is by measuring the height to which the liquid rises in a perfectly clean and vertical capillary tube, the temperature being noted. Quill tubing should be cleaned as described in the last experiment (p. 227), and the cleaned tube used by the student to make his own capillary tubes—a number of which may be prepared and the ends sealed till they are required. After the rise of the liquid in the tube has been determined,—and for this purpose the reading

microscope is most suitable for the wider tubes, though its range is not long enough for the finer tubes, when a cathetometer or millimetre scale to which the tubes may be attached by an elastic band must be used,—the portion of the tube up which the liquid has risen should be broken off and its exact diameter found. Measuring the bore at the upper end by the reading microscope is the quickest way, but the student should satisfy himself that this result is reliable by also measuring one tube (which must be dry and clean) with a thread of mercury (p. 45).

The force which holds up the liquid is the vertical component of the surface tension T, of the liquid, acting along the upper rim of the liquid in the tube; if a be the angle of contact (see next Experiment), this force is therefore $2\pi r \mathrm{T} \cos a$. The weight of the cylindrical thread of liquid sustained in the tube, equivalent to this force, is $\pi r^2 h\rho g$, where h is the mean height of the liquid in the tube in cms. and ρ its density; hence

$$h = \frac{2\mathrm{T}\cos a}{r\rho g}.$$

In those liquids which wet the tube, $\cos a = 1$, we have simply $2\mathrm{T} = rh\rho g$; T being in dynes per lineal cm.

The value $rh\rho$, at any given temperature, is thus a constant for each liquid and is termed its capillary constant.

Example.—Find the capillary constants at 20° C. of water, soap solution, amylic alcohol (fusel oil), and ether.

Enter results thus:—

r and h in cms. and T in dynes per cm., see Tables XXII. and XXIII.

Name of Substance.	ρ.	r.	h.	$rh\rho$.	T.
Water . . .	1·00	·033	4·55	·149	73·1
Soap solution .	1·01	·038	1·32	·051	25·0
Amylic alcohol .	0·82	·035	1·60	·046	22·6
Ether . . .	0·73	·033	1·5	·036	17·6

Exercise.

Find the capillary constants and surface tension of water, soap solution, turpentine, and alcohol.

The size of drops and bubbles also affords another method of measuring the capillary constant of liquids. A convenient method* is to measure the radius of curvature r of a drop hanging in stable equilibrium from a tube, say a millimetre in diameter and under a small definite hydrostatic pressure h, then $2T = rh$. The relative surface tension of liquids may be readily found by comparing the weight of the drops of different liquids issuing under similar conditions from the same orifice, or by counting the number of drops delivered from a pipette, as in the following example. Take a 5 c.c. pipette and bend the delivery tube at right angles. Filled with water at 15° C. it delivered 100 drops; with 1 per cent alcohol mixed with the water it delivered 107

* See Lord Kelvin's *Lectures on the Constitution of Matter*, vol. i. p. 45.

drops; with 2 per cent alcohol 113 drops; with 5 per cent 127; with 10 per cent 145 drops; with 50 per cent 242 drops; the temperature being the same throughout.*

Experiment 91.—**To measure the angle of contact of mercury and glass.**

Instruments required.—Clean mercury, level glass plate, spherometer, and reading microscope.

The immediate cause of the elevation or depression of a liquid in capillary tubes arises from the curvature of the surface due to the surface tension of the liquid in contact with the solid. With water and liquids which wet the tube, the liquid surface inside the tube is always concave outwards; with mercury which does not wet the tube, it is convex, the curvature and therefore the elevation or depression being greater the finer the bore. Owing to surface tension the film tends to reduce the surface to the smallest area within the boundary, that is to a level surface; and so may be considered as pulling the liquid outwards when the surface is concave, and pressing it inwards when the surface is convex, with a force per unit area which is proportional to the tension and the curvature (or sum of the principal curvatures) of the film. Hence immediately under the concave surface the pressure is *less*, and under the convex surface *greater* than the atmosphere by the amount of this force per unit area.†

* See Jamin et Bouty, *Cours de Physique*, vol. i. part ii. p. 65. Obviously this method might be used for commercially testing the alcoholic strength of spirits.

† For a fuller knowledge of this subject the student should especially

If a tangent be drawn to the curved liquid surface where it meets this side of the tube, the angle enclosed between this tangent and the side of the tube is termed the *angle of contact*. This angle has a definite value for each liquid in contact with a particular solid: for pure mercury and clean glass the angle of contact a is about 130°,* if we reckon a as the angle formed by the wedge of liquid in the tube; or the supplement of this, say 40° to 50° (according to different observers), if we measure the external angle, *i.e.* from the tangent to the side of the tube above the liquid. With pure water and clean glass the angle vanishes or becomes equal to 180°.

The value of this angle in the case of mercury may be derived from two measurements—first of the depth of a large drop of mercury on a clean glass plate; then of the capillary depression produced by a clean strip of glass in mercury. Proceed as follows: support a clean plate of glass, say 10 cms. diameter, on a small table provided with levelling screws; after levelling carefully pour on the glass plate some pure clean mercury till a disc of at least 5 or 6 cms. diameter is obtained (the glass plate should rest within a small dish to catch any mercury). Next, by means of the spherometer, find the depth of the disc of mercury; by noting the reflection of the point, or the dimple formed in the liquid surface on screwing

read Professor Tait's lucid exposition in *Properties of Matter*, p. 234, *et seq.*; and Maxwell's *Heat*, chap. xx.

* According to Young the angle of contact of mercury in a barometer tube is 140°. Young determined the angle by the reflection of light from the convex surface of the mercury inside the tube (see *Young's Works*, vol. i., Essays 19 and 20).

down the central point, a very accurate determination can be made. Next pour clean mercury into a small clean glass trough with parallel sides, and hold a narrow strip of clean glass vertically in the trough by means of a clip. With the reading microscope measure the depth of the depression; a careful adjustment of light will be necessary to avoid confusing reflections from the mercury surface. The reading microscope may also be used, instead of the spherometer, to measure the depth of the drop. The angle of contact can then be calculated from the following equations,* where

T = surface tension of mercury in dynes per lineal cm.,

a = external angle of contact,

o = capillary depression in cms.,

e = thickness of a large drop of mercury in cms.,

$$o^2 = \frac{2T(1 - \sin a)}{\rho g},$$

$$e^2 = \frac{2T(1 + \cos a)}{\rho g},$$

whence, $\tan \frac{a}{2} = 1 - \sqrt{2\frac{o}{e}}$, and $T = \frac{o^2 \rho g}{2(1 - \sin a)}$.

Example.—Find the surface tension and angle of contact of mercury with glass.

* Jamin et Bouty, *Cours de Physique*, vol. i. part ii. p. 40.

By measurement of a very large flattened drop of mercury, $o = 0{\cdot}15$ cm., and $e = 0{\cdot}34$ cm., whence as above $a = 41°15'$ and $T = 439{\cdot}6$ dynes per lineal cm.

Exercise.

Repeat the above experiment and calculation.

Note.—The high surface tension of mercury and water renders it extremely difficult to obtain, or to keep, a clean surface to these liquids; in the last experiment the mercury was not quite clean. Dipping the finger into water instantly contaminates the surface with a greasy film and lowers the surface tension. Lord Rayleigh has shown that distilled water poured from a stock bottle gathers impurities from the side of the bottle, and has a dirtier surface than water drawn from a tap. The easiest way to obtain a fairly clean surface of water or mercury is to draw off the liquids through a tube that dips below their surface, rejecting the first portions; a wash bottle well cleansed answers the purpose. More perfect methods of obtaining a clean water surface are described in Appendix, § 17. The activity of fragments of camphor affords a test of a moderately clean water surface, and the absence of "surface viscosity" in water is a test of perfect cleanliness.

APPENDIX

§ 1. **Theory of the vernier.**

If n divisions on the vernier be equal to $n+1$ or $n-1$ divisions on the scale; and if S be the length of a scale division and V the length of a vernier division, then

$$(n+1)S = nV,$$

i.e

$$V = \frac{n+1}{n}S;$$

$$\therefore\ V - S = \frac{n+1}{n}S - S = \frac{1}{n}S, \quad \therefore\ S = n(V - S) \quad . \quad (\alpha).$$

Also

$$(n-1)S = nV,$$

i.e.

$$V = \frac{n-1}{n}S;$$

$$\therefore\ S - V = S - \frac{n-1}{n}S = \frac{1}{n}S, \quad \therefore\ n(S - V) = S \quad . \quad (\beta).$$

$\frac{1}{n}S$ is the least count of the *vernier*.

§ 2. **Theory of the spherometer.**

In Fig. 61 A, B, C are the positions of the three fixed feet of the spherometer, and O that of the movable foot, when they are all in the same plane.

$$AB = l, \quad \text{and angle } ABC = 60°,$$

$$\therefore\ AD = l \sin 60° = l\frac{\sqrt{3}}{2},$$

and

$$AO = \frac{2}{3}AD = \frac{2}{3} \times l\frac{\sqrt{3}}{2} = \frac{l}{\sqrt{3}}.$$

In Fig. 62 $FK \times KE = \overline{KH}^2$, but $FK = FE - EK = 2R - a$, and AO in Fig. 61 is the same as KH in Fig. 62;

$$\therefore \ (2R - a)a = \overline{AO}^2 = \frac{l^2}{3},$$

and
$$R = \frac{l^2}{6a} + \frac{a}{2}.$$

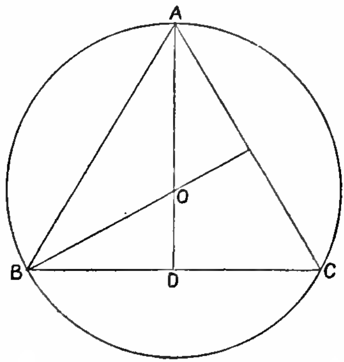


Fig. 61.

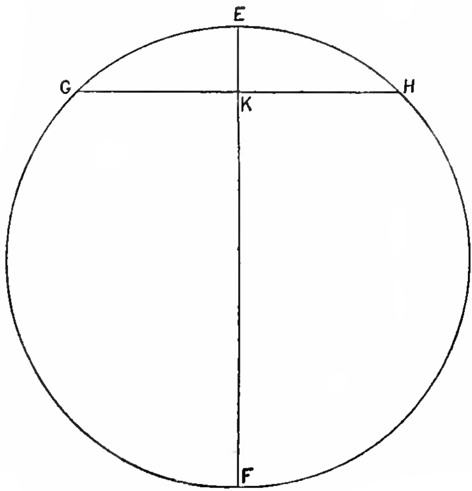


Fig. 62.

§ 3. **Reading microscope and telescope.**

The cathetometer is a costly and often defective piece of physical apparatus; unless well made it is seldom reliable to the "least count" of its readings, which may therefore give a misleading appearance of accuracy. A simpler and more trustworthy method of measuring small differences of length is to employ a microscope, in the eye-piece of which is a transparent micro-photograph of a well-divided scale. The value of each division can be ascertained by focusing on a millimetre rule; 10 or 20 divisions of the micro-photograph to a millimetre can easily be seen, and reliance can therefore be placed on readings to the $\frac{1}{20}$ of a mm. This method has the advantage of enabling readings to be taken more quickly than with the cathetometer, is a much less costly arrangement, and more reliable. Moreover, by merely turning the eye-piece, horizontal as well as vertical displacements may be read. Obviously, however, its use is limited to observations where the total space to be measured is small. Quincke's *cathetometer microscope* is an instrument of this kind; it is mounted on a small glass table with levelling screws.

To enable longer vertical spaces to be measured a *reading telescope* may be employed. This is simply an ordinary telescope capable of being focused upon objects within 10 or 15 feet distant, and having a cross wire in the focus of the eye-piece. The telescope slides on a vertical rod, to which it can be clamped at any height. A millimetre scale is placed beside the object to be measured, the cross wire brought to coincide with the upper mark, the corresponding scale reading taken, and then the telescope is slid down to the lower mark and the scale reading again taken; the difference of the two readings gives the space between the marks. Better results will often be given with this method than by a cathetometer, as small differences in the level of the telescope do not destroy the accuracy of the result, as with the cathetometer.

The *reading microscope* is one of the most accurate and convenient instruments for measuring small quantities, such as the linear coefficient of expansion of a rod by heat, the

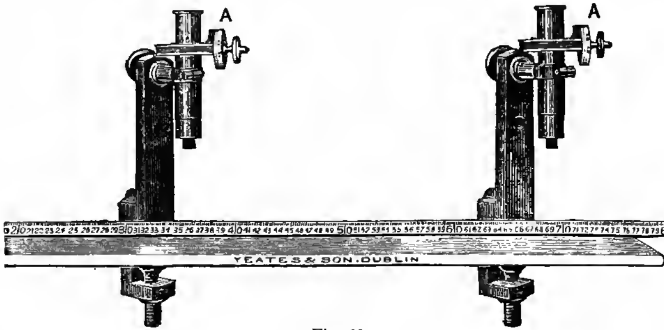


Fig. 63.

diameter of a wire or the bore of a capillary tube, if of circular section. Two reading microscopes sliding on a firm bed, to which they can be clamped (Fig. 63), form an accurate optical beam compass, and are thus used for determining the linear coefficient of expansion, etc. This arrangement will be more fully described in the next volume. In the eye-piece of the microscope is a single fine hair or wire, stretched on a frame which can be moved across the field of view by a fine screw attached to a divided circle A. Fixed in the eye-piece is a scale (shown at C in Fig. 64), the distance between each of its divisions being equal to the pitch of the screw, hence a complete rotation of the circle A (usually divided into 100 parts) moves the hair line H (Fig. 64) through one division of the fixed scale. To find the value of one division the instrument is focused on a millimetre scale, which is

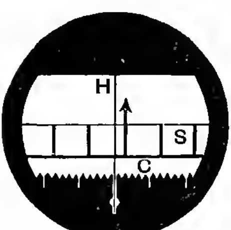


Fig. 64.

shown magnified in the field of view, at S (Fig. 63), where five divisions of the fixed scale C are seen to coincide with 1 millimetre, hence each division = 0·2 mm., and as the divided circle enables $\frac{1}{100}$ of this space to be read, the instrument can measure to 0·002 of a millimetre.

§ 4. **The so-called horizontal or micrometer-pendulum.** This instrument was first devised, in 1832, by Hengeller (*Phil. Mag.* vol. xlvi. p. 416), then independently by the Rev. M. H. Close, of Dublin, and a little later by Zöllner.

The instrument here described is the form devised by Mr. Close, the mode of suspension being different from and preferable to Hengeller's or Zöllner's. It consists essentially of a rod AB supported horizontally for convenience by two silk threads AC and BD, as shown in Fig. 65, so that the rod oscillates about the axis CD. The threads should be as fine as possible, so that their stiffness should interfere as little as may be with the behaviour of the pendulum.

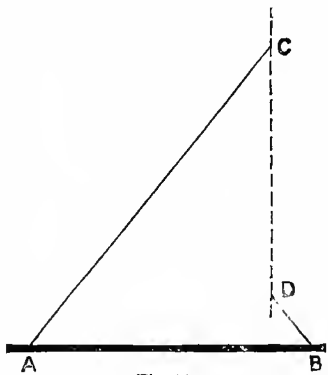


Fig. 65.

In order that the rod AB may be in stable equilibrium the axis CD is inclined slightly towards A. If its inclination θ to the vertical be very small, and if we could neglect the stiffness of the fibres at C and D, then an exceedingly small tilt β of the axis CD at right angles to the plane of rest of AB would produce an angular movement of the pendulum equal to $\beta/\sin\theta$. Thus if θ were $1'$, the magnifying power of the pendulum would be $1/\sin 1'$ or 3438. The stiffness of the

threads makes this less—how much can only be determined by direct experiment.

Obviously the best way of using the pendulum is to attach a mirror to the rod, and view either a reflected scale or a reflected spot of light. In this way the pendulum rod may be kept as short as desirable, and the sensitiveness enormously increased.

The instrument is mounted on a firm base with two fine graduated levelling screws: one to regulate the inclination of CD towards A, and thus to modify the sensitiveness of the instrument as desired; the other to tilt the axis in a direction at right angles to the vertical plane of rest of the pendulum, in order to bring the pendulum, when necessary, to its zero point, and also to measure its delicacy by observing what angular movement would be produced in it by a given transverse tilt.

The micrometer-pendulum can be easily made by the student, and so delicate is the instrument that it will be found necessary to place it on a support free from vibration, and to enclose the whole in a box, with glass front, lined with tinfoil joined to earth, to prevent any electrical or air disturbance. The pendulum rod also should not be made of any magnetic metal; a stout brass or platinum wire answers very well.

Mr. Close has found that in a moderate gale, the micrometer-pendulum, placed on a rigid support in the basement of a house, shows that at every gust of wind the whole ground floor of the house tilts over to leeward through an angle considerably greater than can be measured by the micrometer-pendulum when it is finely adjusted.

§ 5. **Theory of the balance in the case when the points of suspension of the beam and the points of support for the scale pans are all in the one straight line.**

In Fig. 66 let

W = weight of the balance beam,

l = CB the half length of the beam,

l' = CG the distance from the point of suspension to the centre of gravity of the beam,

P = weight in one pan,

$P+p$ = weight in other pan,

θ = the angle which the beam is turned through by the weight p.

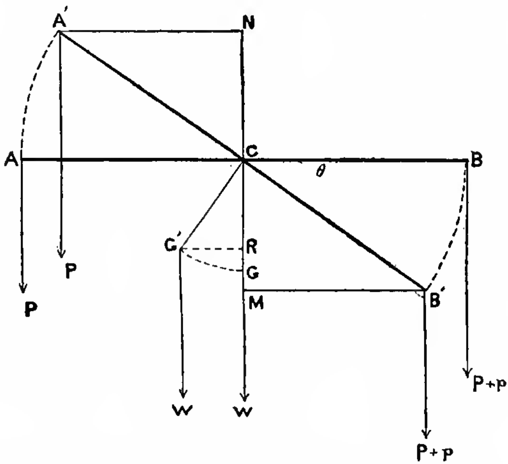


Fig. 66.

Now taking moments round C we get

$$(P+p)B'M = W \times G'R + P \times A'N,$$

$$(P+p)l \cos\theta = Wl' \sin\theta + Pl \cos\theta,$$

$$pl \cos\theta = Wl' \sin\theta,$$

$$\tan\theta = \frac{pl}{Wl'},$$

which expresses the sensibility of the balance.

§ 6. **Specific gravity—Hydrometers.**

(1) For the density of an insoluble body lighter than water let

W = weight of the *solid* in air,
W^1 = „ *sinker* „
W_1 = „ „ in water,
W_2 = „ solid and sinker together in water,

then $W^1 - W_1$ = weight of water displaced by the sinker alone,

and $(W + W^1) - W_2$ = weight of water displaced by both sinker and body;

hence $(W + W^1 - W_2) - (W^1 - W_1) = W + W_1 - W_2$ is the weight of water displaced by the body alone;

$$\therefore \ \rho = \frac{W}{W + W_1 - W_2}.$$

(2) All hydrometers are subject to a slight error, arising from the surface tension of the liquid in which they float; this causes a film of the liquid to be drawn up the stem, and the hydrometer floats at a higher position than it otherwise would do; this would not matter if all liquids had the same surface tension, but this is not the case. The error due to this cause may be reduced by making the stem of the hydrometer very thin, as in Nicholson's hydrometer, and if, before use, the stem be wiped with a clean dry cloth or with a cloth damped with alcohol, the error is still further reduced. Mr. Joly, F.R.S. (*Proc. Royal Dublin Soc.*, 1886) has devised a form of Nicholson's hydrometer which is practically free from this error, and at the same time forms a delicate balance for ordinary purposes. Joly's hydrometer is a suspended metal globe, made in two pieces, having a narrow tubulure below; the globe is filled with water, and within it floats a smaller empty metal or glass globe from which depends a fine wire passing through the tubulure and carrying a pan below. The instrument is therefore an inverted Nicholson, with the weights

acting in tension and not in compression on the fine wire stem. Owing to the narrowness of the tubulure the water is unable to escape from the globe. The outer globe if of metal may be made in two hemispheres, fastened together after the inner globe has been inserted; or if of glass the inner globe can be blown inside it and a brass cap and narrow tube for the wire cemented on the neck.

(3) Mr. Joly has also devised a method for determining the specific gravity of minute specimens,* which has many advantages: it is simple, accurate, expeditious, and can be applied to porous bodies. The method consists in embedding the specimen in a little disc of paraffin wax, cut from a paraffin candle, about 1·5 mm. thick and 3 or 4 mms. diameter, the edges smoothed and the size of the disc adapted to the quantity of the solid to be tested. The disc is carefully weighed, and the fragment or fragments to be tested placed on the paraffin and bedded in it by holding a hot copper wire over the specimen; the compound body or pellet is again weighed, the increase in weight gives the weight of the specimen. A saturated solution of common salt and water having been prepared, the pellet is dropped in and water added till the pellet is just balanced in the liquid; the final adjustment and stirring of the liquid is best accomplished by a camel's-hair brush. By a Sprengel's tube or hydrometer, the specific gravity of the solution is then found which gives the specific gravity of the pellet of wax enclosing the specimen; from this the specific gravity of the specimen can be found, the specific gravity of paraffin wax having been previously determined. If

W = weight of the specimen in air,
w = ,, ,, paraffin used,
σ = specific gravity of the paraffin used,
s = ,, ,, mixed substance,

* *Proc. Royal Dublin Soc.*, January 1886.

then the specific gravity of the specimen $\rho = \dfrac{W}{\dfrac{W+w}{s} - \dfrac{w}{\sigma}}$

or $\rho = \dfrac{W\sigma s}{(W+w)\sigma - ws}$.

Example.—A specimen of magnetite from the Krakatoa ash bedded in paraffin.

$$W = 0{\cdot}0202 \text{ grm.}, \quad w = 0{\cdot}0654 \text{ grm.},$$
$$\sigma = 0{\cdot}9206 \;\;\text{,,} \qquad s = 1{\cdot}1361 \;\;\text{,,}$$
$$\therefore \; \rho = 4{\cdot}71.$$

One great advantage of this method is the complete extrusion of the entangled air by the melted paraffin soaking into the specimen, and, moreover, a specimen of any density above unity can be determined, the method described on p. 72 being limited to bodies below 3·4.

§ 7. Correction of the height of the barometric column for temperature.

Let

V_t and V_o = volume of the mercury at $t°$ C. and 0° C. respectively,

h_t and h_o = height of mercury column at $t°$ C. and 0° C. respectively,

δ = coefficient of cubical expansion of mercury,

α = coefficient of linear expansion of brass,

H = true corrected height of mercury column,

then
$$h_o = h_t \frac{V_o}{V_t}, \qquad \text{and } V_t = V_o(1 + \delta t),$$

$$\therefore \; \frac{V_o}{V_t} = \frac{1}{1+\delta t} = 1 - \delta t + \delta^2 t^2, \text{ etc.,}$$

and by neglecting $\delta^2 t^2$ and following factors, we have

$$h_o = h_t(1 - \delta t).$$

In the same way the length of the column will be *apparently* increased by the expansion of the scale in the ratio of $1 + at$ to 1,

$$\therefore \mathrm{H} = h_o(1 + at) = h_t(1 + at)(1 - \delta t)$$
$$\doteq h_t\{1 - (\delta - a)t\},$$

and since
$$\delta = 0{\cdot}000182,$$
$$a = 0{\cdot}000020,$$
$$\therefore \mathrm{H} = h_t(1 - 0{\cdot}000162t).$$

§ 8. **Joly's mercury-glycerine barometer.**

Various devices have been made to increase the sensitiveness of a mercurial barometer, but what is gained in an open scale or long range is usually lost by increased friction and irregularity of action. The common weather glass or wheel barometer is an illustration of this. A simpler plan is to bend the upper part of a long barometer tube to an obtuse angle; a vertical rise or fall of an inch will thus cause the mercury to move over a foot or more of the inclined tube, but the same objection applies. Using a liquid of less density than mercury enables a proportionally longer range to be obtained; thus in a water barometer the range is obviously 13½ times that of a mercurial barometer, but the tube has to be some 35 feet long, and the considerable vapour pressure of water, variable with temperature, destroys its usefulness. Glycerine is a far better liquid to use, owing to its non-volatility, but the difficulty of construction and inconvenient length of the tube is a drawback.

Mr. Joly's device enables a glycerine barometer to be made of moderate length.* As the arrangement may be useful for laboratory purposes the following details are given. A plug or disc of ebonite or ivory, having a diameter a little less than the barometer tube, is made with a steel pin projecting from it, a small sphere or cylinder of wood or ivory called the float

* *Proc. Royal Dublin Soc.*, 1892.

being fastened to the end of the pin, as shown at B, Fig. 67. Placed at the bottom of a column of mercury, the float tends to rise with a force equal to the difference in weight of the float and the mercury it displaces. If, however, the float be made of the right bulk, it will be unable to pull the plug after it through the mercury. The lower surface of the mercury being covered by the disc or plug remains unbroken by the weight of the mercury column above, and through the very narrow annular space around the plug the mercury does not pass. A column of mercury may thus be sustained in a wide tube closed at the upper end only; moreover, the elasticity of the air permits the column to be oscillated with considerable violence without any mercury escaping or air creeping up through it. The space below the mercury may be filled with glycerine, and the tube may dip into an open vessel of glycerine; the mercury will thus appear to float on the glycerine, and the latter will enter or leave the tube as the mercury rises or falls. If the space above the mercury be a vacuum and the mercury column say 27 inches long, the glycerine column below the tube must be of such a length that the joint pressure of the mercury and glycerine shall be equal to the atmospheric pressure. If now the atmospheric pressure rise or fall, as much glycerine must enter or leave the tube as if it were a glycerine barometer of corresponding bore. Hence the range of the instrument is the same as that of a glycerine barometer, though its length need not be more than a few feet, whilst its sensitiveness and promptness is probably greater, as there is a much shorter length of the viscous glycerine to move. It is important that the bore of the tube be uniform, or the scale will be incorrect; if not quite uniform the tube should be calibrated and a corrected scale applied.

Fig. 67.

The tube is filled as follows: A glass tube about $\frac{1}{2}$ or $\frac{3}{4}$ of an inch in diameter, and say 8 feet long, has one end closed and filled to a height of some 27 inches with clean, dry, warm mercury; adhering bubbles of air may be removed by tapping (see p. 96) or by means of an air-pump. An ebonite plug or cylinder (A), with its length equal to its diameter, which latter should be nearly equal to that of the tube, is now dropped in. The float (B) attached to the plug may be a small sphere of ebonite, as in the figure, which is drawn to scale; or for a barometer tube $\frac{3}{4}$ of an inch diameter an ebonite cylinder $\frac{1}{2}$ an inch long and $\frac{1}{4}$ of an inch diameter will be found suitable.* Mercury is then poured on the plug till the float is completely submerged. Glycerine that has been previously warmed and its dissolved air extracted by an air-pump is now poured in. When the tube is full it is carefully corked, so that no air bubble is entrapped, and then the tube is inverted in a bath of glycerine and the cork cautiously withdrawn. The column sinks slowly until the atmospheric pressure is balanced, when the operation is complete. The bath may be a glass dish about 4 inches deep and 8 inches diameter, though a larger rectangular trough would be better. A wooden cover excludes dust, and a little sperm oil floating on the surface of the glycerine in the bath prevents absorption of moisture from the atmosphere. If the bath be placed on the floor, the top of the column is about level with the eye and thus convenient to read. The scale must be set by an ordinary barometer, and the graduation made from the density, say 1·26, of the glycerine employed; about 1·1 inch would therefore be equivalent to one-tenth of an inch in a mercurial barometer. Creosote, with some advantage, can be used in place of glycerine.

* The proportion of the float to the plug is important; if too large it will drag up the plug. The plug A must be of sufficient depth; if too shallow it will be unstable and apt to jam in the tube.

§ 9. The measurement of small fluid pressures.

The M'Leod gauge is subject to an error arising from the condensation of the residual gas on the surface of the glass gauge tube and bulb; with gases like CO_2 this condensation is considerable, and vitiates the indications given by the instrument. By cautiously heating the glass the condensed gases may be expelled, but, unless removed, the residual gas will be re-absorbed on the glass cooling. The highest vacuum may, however, be obtained by making the residual gas CO_2, heating the vessel under exhaustion whilst the pump is at work, and absorbing the last traces of gas by fused caustic potash. When the vacuum is very good the spark from an induction coil ceases to pass between two adjacent platinum wires fused into the vessel under exhaustion.

The difficulty of measuring small pressures arises less from the smallness of the force (as a millionth of an atmosphere is equal to the weight of a milligramme per sq. cm.) than from the difficulty of making the surface pressed on, such as a liquid in a U-tube, freely movable. The best method appears to be to observe through a microscope the motion of a thin flat film or membrane, such as glass, mica, or india-rubber, stretched across a tube and subject to a small difference of pressure on its opposite faces. In this way, as Fitzgerald, to whom this method is due, suggests, the relative densities of gases may be measured by balancing one column of gas against another; the density thus found would be free from the error arising from the condensation of the gas on the surface of the vessel in which it is weighed.

§ 10. Proof of the formula for the Atwood's machine.

Let

g = acceleration due to gravity,

a = acceleration produced by the mass m,

S = distance fallen *before* m is removed,

S_1 = distance fallen *after* m is removed,
t = time taken to fall through S,
t_1 = " " S_1,
K = a constant, equivalent to the moment of inertia of the wheel-work,
w = a mass whose weight is equivalent to friction,

then $\qquad 2M + m + K$ may be called the mass moved,

$$\therefore\ (2M + m + K)a = (m - w)g,$$

$$\therefore\quad a = \frac{(m - w)g}{2M + m + K};$$

also $\qquad S = \frac{1}{2}at^2.$

And if v be the velocity at the instant that m is removed,

$$v^2 = 2aS,$$

$$S_1 = vt_1,$$

$$\therefore\ S_1^{\ 2} = v^2t_1^{\ 2} = 2aSt_1^{\ 2},$$

$$a = \frac{S_1^{\ 2}}{2St_1^{\ 2}} = \frac{(m - w)g}{2M + m + K},$$

$$\frac{2St_1^{\ 2}(m - w)g}{S_1^{\ 2}} = 2M + m + K \quad . \quad . \quad (1).$$

Now by using a heavier rider m', and keeping the distances the same, we also get

$$\frac{2St_2^{\ 2}(m' - w)g}{S_1^{\ 2}} = 2M + m' + K \quad . \quad . \quad (2),$$

and by subtracting the former of these two equations from the latter to eliminate K, we arrive at the equation in the text.

§ 11. **Proof of the simple pendulum formula to the first approximation.**

(1) To find the acceleration of a body towards the centre of a circle.

In Fig. 68 if a body be moving in a circle with uniform

velocity V, and P be the position of the body at any instant, if no force act upon it, in time t it would be at M, when $PM = Vt$.

Therefore MN represents the distance through which

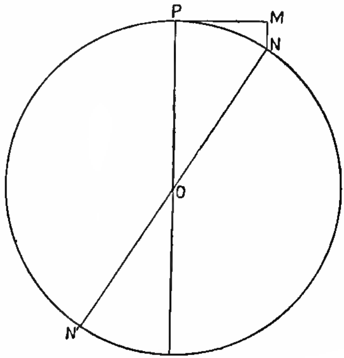


Fig. 68.

the body has been moved by the central force; and if a be the acceleration towards the centre, $MN = \frac{1}{2}at^2$.

But $PM^2 = 2OP \times MN$ in the limit (*Euclid* III. 37),

$$\therefore \quad V^2t^2 = 2r \times \tfrac{1}{2}at^2,$$

$$\therefore \quad a = \frac{V^2}{r}.$$

(2) To find the relation between *acceleration* and *displacement* in simple harmonic motion.

In Fig. 69 a point P moving uniformly in a circle with velocity V, the foot M of the perpendicular from P on the diameter AA′ will move to and fro along AA′, and this motion of M is called simple harmonic motion. If

$$OP = r, \quad OM = d, \quad \text{angle } POM = \theta,$$

then from (1) the acceleration of P towards the centre is

$a = \frac{V^2}{r}$, and a the component of this along AA′ or the acceleration of M is $\frac{V^2}{r} \cos \theta = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} \cos \theta$, if T be the periodic time of P.

And the displacement of M from O is $d = r \cos \theta$,

$$\therefore \frac{a}{d} = \frac{4\pi^2 r^2}{T^2 r} \cos \theta \Big/ r \cos \theta = \frac{4\pi^2}{T^2}.$$

(For harmonic motion, see Daniell's *Physics*, chap. v., or Everett's *Vibratory Motion and Sound.*)

(3) To find the relation between the acceleration and displacement in a simple pendulum.

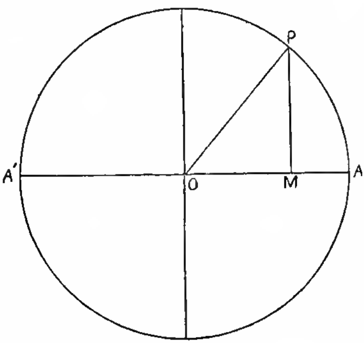


Fig. 69.

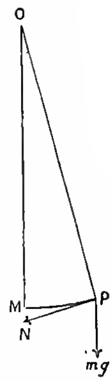


Fig. 70.

Let l = length of the pendulum,
T = the periodic time of a *vibration*,
g = acceleration due to gravity.

In the simple pendulum (Fig. 70) $OP = l$ and the angle $POM = \theta$ the angle of displacement, and $PM = l\theta = d$ the linear displacement, the effective acceleration a is in the direction PN, and equal to $g \sin \theta$;

$$\therefore \frac{a}{d} = \frac{g \sin \theta}{l\theta} = \frac{g}{l} \text{ (θ being very small).}$$

Therefore, for a small displacement, the motion of the bob of a simple pendulum is very approximately a simple harmonic motion, and for a point performing such a motion we have seen in (2) that $\frac{a}{d}=\frac{4\pi^2}{T^2}$.

Therefore from (3) we get

$$\frac{4\pi^2}{T^2}=\frac{g}{l}; \qquad \therefore\ T=2\pi\sqrt{\frac{l}{g}}.$$

T being the periodic time of a vibration, an oscillation or single swing is $t=\pi\sqrt{\frac{l}{g}}$.

For a nearer approximation $T=2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\theta}{2}\right)$.

§ 12. **To find the length of a simple pendulum whose time of oscillation is equal to that of a given compound pendulum.**

This can be done by experiment, as described in the middle paragraph of p. 129; or if the bob of the pendulum be a sphere of uniform density and radius r, the length l of the pendulum from the centre of the sphere to the point of suspension must be increased by $\frac{2}{5}\cdot\frac{r^2}{l}$, so that the corrected length becomes $l+\frac{2r^2}{5l}$. (See moment of inertia of a sphere, Table XIV., and Hicks' *Dynamics*, p. 347.)

§ 13. **Proof of the formula for the Ballistic pendulum.** Let

M = mass of the bob of the pendulum with the bullet in it,

m = mass of the bullet,

h = distance from the centre of inertia of the bob to the knife-edge,
h_1 = distance of the line of fire from the knife-edge,
h_2 = distance which the centre of inertia of the bob rises by impact,
L = distance of attachment of tape from the knife-edge,
S = length of tape drawn out,
T = half period of equivalent simple pendulum,
V = velocity of bullet just before impact,
ω = angular velocity of pendulum when impact is complete,
θ = angle of displacement,
K = radius of gyration of pendulum,
g = acceleration due to gravity.

Now, when a body is struck the change of its moment of momentum about any axis is equal to the moment of the impact about the same axis; or the whole moment of momentum about the axis of the pendulum is the same before impact and after it.

$$\therefore\ m\mathrm{V}h_1 = \mathrm{MK}^2\omega,\ \therefore\ \omega = \frac{m\mathrm{V}h_1}{\mathrm{MK}^2}.$$

And when the deflection of the bob is greatest its kinetic energy is changed into its equivalent potential energy, and since $h_2 = h\ (1 - \cos\ \theta)$ we have

$$\mathrm{M}gh(1 - \cos\ \theta) = \tfrac{1}{2}\mathrm{MK}^2\omega^2 = \tfrac{1}{2}\frac{m^2\mathrm{V}^2h_1^{\ 2}}{\mathrm{MK}^2}.$$

By substituting the value of ω above,

i.e. $$\frac{m^2\mathrm{V}^2h_1^{\ 2}}{\mathrm{MK}^2} = \mathrm{M}gh\,4\,\sin^2\frac{\theta}{2};$$

$$\therefore\ \mathrm{V} = \frac{\mathrm{MK} \times 2\ \sin\ \dfrac{\theta}{2} \times \sqrt{gh}}{mh_1}.$$

But $$2 \sin \frac{\theta}{2} = \frac{S}{L}, \quad \text{and} \quad T = \pi \sqrt{\frac{K^2}{gh}},$$

$\therefore \ K = \dfrac{T\sqrt{gh}}{\pi}$. Substituting we get

$$V = \frac{MhgTS}{mh_1 L\pi}.$$

(See Thomson and Tait's *Natural Philosophy*, small edition, p. 91.)

§ 14. **To find the moment of inertia of a vibrator; for example a right cylinder with the axis of figure the axis of rotation.**

Let $I =$ moment of inertia of cylinder,

$M =$ mass,

$l =$ length,

$R =$ radius,

$\rho =$ density.

Suppose an infinitely thin cylinder of radius r and mass dm, its moment of inertia with respect to the axis is $r^2 dm$; and $dm = 2\pi r l \rho dr$.

And considering each thin cylinder as an element of the whole cylinder, we have

$$I = 2\pi l\rho \int_0^R r^3 dr = \tfrac{1}{2}\pi l \rho R^4;$$

and since $M = \pi R^2 l \rho$,

$$I = \tfrac{1}{2} M R^2.$$

§ 15. **Theory of simple rigidity.**

A wire is fixed firmly at one end to a support and hung vertically downwards with a vibrator at the other end.

Let

n = simple rigidity,

T = moment of torsional couple due to unit twist on the wire,

I = moment of inertia of vibrator,

t = half period of vibrator,

l = length of wire,

r = radius of wire.

Now if x be the distance from the centre of the wire to an elementary ring of breadth dx, the area of this ring will be

$$\pi x^2 - \pi(x+dx)^2 = 2\pi x dx;$$

by neglecting πdx^2, and when the vibrator is twisted through unit angle or one radian, we have

$$\frac{x}{l} = \text{shearing strain at a point in the ring,}$$

and since n = stress/strain,

$$\frac{nx}{l} = \text{stress per unit area at distance } x \text{ from the centre;}$$

$$\therefore \frac{2\pi n x^2 dx}{l} = \text{stress for the whole elementary area,}$$

and $$\frac{2\pi n x^3 dx}{l} = \text{moment of this stress;}$$

$$\therefore \mathrm{T} = \frac{2\pi n}{l}\int_0^r x^3 dx = \frac{n\pi r^4}{2l},$$

which is the moment of the stress for the whole section.

But $$t = \pi\sqrt{\frac{\mathrm{I}}{\mathrm{T}}}; \quad \therefore \mathrm{T} = \frac{\pi^2 \mathrm{I}}{t^2},$$

and by substitution we get $\dfrac{n\pi r^4}{2l} = \dfrac{\pi^2\mathrm{I}}{t^2}$;

$$\therefore n = \frac{2\pi l \mathrm{I}}{r^4 t^2}.$$

§ 16. **Effusion, transpiration, and diffusion of gases.**

The passage of gases through (i.) a fine hole in a thin plate, *effusion*, or (ii.) through capillary tubes, *transpiration*, or (iii.) through a porous solid, *diffusion*, are distinct processes, as first pointed out by Graham. Though the rate of effusion and diffusion both vary inversely as the square root of the densities of the gases, yet they are processes essentially different in their nature. When the aperture of efflux becomes a tube the effusion rate is disturbed, and becomes the transpiration rate when the length of the capillary tube exceeds its diameter by at least 4000 times. The rates of transpiration are singularly unlike the rates of effusion, as will be seen by comparing Tables XX. and XXI.; note, however, that in Table XX. *times* of transpiration are given which are of course inversely as the rates. A plate of compressed plumbago or of "biscuitware," though impermeable to gas by effusion or transpiration, is readily penetrated by the molecular or diffusive movement of gases. Through such a plate a separation of mixed gases follows as a consequence of the movement being molecular, thus a mixture of 67 of H_2 and 33 of O_2 before diffusion became 9 of H_2 and 91 of O_2 after diffusion. This gaseous analysis by diffusion has been termed *atmolysis*. A very different behaviour occurs if different gases are separated by a membrane, such as caoutchouc, in which the gas may be more or less soluble.

The kinetic theory of gases gives, as Clerk Maxwell points out (*Heat*, new ed., p. 331), a simple relation between the diffusion, viscosity, and conductivity of heat in gases; these phenomena express "the rate of equalisation of three properties of the medium,—the proportion of its ingredients, its velocity, and its temperature. The equalisation is effected by the same agency in each case, namely, the agitation of the molecules. In each case, if the density remains the same, the rate of equalisation is proportional to the absolute temperature."

Recent experiments have shown that the exact law of the variation of the viscosity of gases with temperature is still uncertain.

§ 17. **Removal of surface contamination from water.**

In order to obtain a perfectly clean surface on water the following procedure, due to Lord Rayleigh, is necessary. Draw off water from a tap, or distilled water from a tube dipping below the surface, reject the first portions, receive the water in a well cleansed tin tray. Now bend a long strip of thin clean sheet-brass, about two inches wide, into as small a hoop as possible; dip its edge just below the surface of the water and open out the hoop. By this means the surface contamination will be swept aside, and will take some time to return; repeat this process two or three times, each time the contamination increases on the outside and diminishes within the hoop. Finally, by means of a Bunsen flame held beneath the tin tray, slightly warm the cleansed portion of the water; by this means the colder water surrounding will have a relatively higher surface tension, and draw off the remaining trace of any greasy film. A gentle current of air along the surface may be used to cleanse the surface where the vessel containing the water cannot be warmed. The student should try the capillary constant of water in such a cleansed surface (see Table XXIII.).

The activity of the motion of fragments of camphor dropped on the surface of water is a convenient test for a moderately clean surface (p. 236). Lord Rayleigh has shown that a greasy film two millionths of a millimetre ($2\ \mu\mu$) in thickness will arrest these movements; the black and thinnest parts of a soap bubble are about $12\ \mu\mu$. Still thinner probably, about $\frac{1}{10}\ \mu\mu$, is the film which gives rise to the effects of "surface viscosity."

§ 18. **Superficial viscosity, formation of soap films.**

On the surface of ordinary water a vibrating compass needle comes to rest in half the time it requires when vibrating wholly within the water. This apparent surface viscosity is caused by the oscillating needle sweeping the contamination on the surface before its advancing edge, and therefore leaving less behind it. The tension is consequently higher behind than in front of the moving needle, hence a force comes into play which damps the vibration. This quasi-surface viscosity disappears when the water surface is cleansed, as described in § 17. Motes or sulphur dusted on the surface of water reveal the apparent viscosity of the surface; on an unprepared surface of water a vibrating compass needle will be seen to set the whole in motion, but with a surface freed from impurity the motes do not move until the vibrating needle almost strikes them.

Lord Rayleigh's investigations render it highly probable that the power, which soap confers upon water, of forming a fairly durable bubble and extending into a stable film, is really due to the creation of an outer coating or pellicle, the surface tension of which is less than that of the purer water within. "The stability of the film requires that the tension be not absolutely constant, but liable to augment under extension. If the central parts of a vertical film were suddenly displaced downwards, an increase of tension above and a decrease below would be called into play, and the original condition would be restored" (Rayleigh). The stronger tension of the purer water in the innermost part of a soap film not only tends to resist the rupture of the outer pellicle, but helps to repair it when broken and to pull it up against the force of gravity, which is thinning a vertical film. A self-acting adjustment therefore comes into play preserving the equilibrium and creating the comparative stability of a soap film. The behaviour of oil or turpentine on water (see Experiment 88, p. 226) illustrates what takes place.

§ 19. **Theory of the rise of liquids in capillary tubes.**

The capillary tube is placed vertically in a vessel containing liquid. Let

h = height from the lower part of the meniscus to the level of the liquid in the vessel,
r = radius of the tube,
T = surface tension of the liquid acting at angle θ to the vertical,
θ = capillary angle,
ρ = density of the liquid,

then $\mathrm{T}\cos\theta$ = vertical component of T,

$2\pi r\mathrm{T}\cos\theta$ = total force tending to raise the liquid in the tube,

and, neglecting the liquid forming the meniscus, the force in absolute units required to support the liquid in the tube is

$$h\pi r^2\rho g.$$

Therefore for equilibrium we have

$$2\pi r\mathrm{T}\cos\theta = h\pi r^2\rho g,$$

$$\therefore\ \mathrm{T} = \frac{hr\rho g}{2\cos\theta},$$

and
$$hr = \frac{2\mathrm{T}\cos\theta}{\rho g} = \text{a constant.}$$

If the liquid wets the tube, we have approximately $\theta = 0$ and $\cos\theta = 1$,

$$\therefore\ hr = \frac{2\mathrm{T}}{\rho g} = \text{a constant.}$$

§ 20. **Soap solution for soap films.**

The following method of preparing a good solution for soap films is described and recommended by Mr. Vernon Boys :—

"Fill a clean stoppered-bottle three-quarters full of water. Add one-fortieth part of its weight of oleate of soda, which

will probably float on the water. Leave it for a day, when the oleate of soda will be dissolved. Nearly fill up the bottle with Price's glycerine and shake well, or pour it into another clean bottle and back again several times. Leave the bottle, stoppered of course, for about a week in a dark place. Then with a syphon draw off the clear liquid from the scum which will have collected at the top. Add one or two drops of strong liquid ammonia to every pint of the liquid. Then carefully keep it in a stoppered bottle in a dark place. Do not get out this stock bottle every time a bubble is to be blown, but have a small working bottle; never put any back into the stock. In making the liquid *do not warm or filter it*; either will spoil it. Never leave the stopper out of the bottle, nor allow the liquid to be exposed to the air more than is necessary. This liquid is still perfectly good after two years' keeping." The oleate of soda should be fresh, if dry it is not so good; when it cannot be obtained, Castille or Marseilles soap, obtainable at any chemists, answers almost as well.

In the *Philosophical Magazine*, January 1867, p. 40 *et seq.*, Plateau describes the preparation of his soap-glycerine solution, bubbles blown with which have a great permanence; a bubble 1 decimetre in diameter, supported on a wire ring, lasted 24 hours when freely exposed to the air, and upwards of 54 hours when covered by a vessel containing *dry* air; a soap film, 7 cms. diameter, formed in a bottle of the solution, lasted 18 days.

TABLES

Table I.

Units and Dimensions.

(The numbers in the table are the indices of [L] [M] [T], thus the dimensions of force are [L] [M] [T^{-2}]).

Derived Units.	Fundamental Units.		
	Length [L.]	Mass [M.]	Time [T.]
Surface	2	0	0
Volume	3	0	0
Angle	0	0	0
Velocity	1	0	− 1
Angular velocity . .	0	0	− 1
Acceleration . . .	1	0	− 2
Force	1	1	− 2
Pressure . . .	− 2	1	− 2
Density	− 3	1	0
Momentum . .	1	1	− 1
Work . . .	2	1	− 2
Moment of inertia . .	2	1	0
Moment of momentum .	2	1	− 1

Table II.

Metrical and British Measurements.

(Chiefly from Everett's *Units and Physical Constants.*)

1 Metre	= 1·094 yards.
1 Metre	= 3·281 feet.
1 Metre	= 39·3704 inches.
1 Micron (μ)	= one thousandth of a mm. = 10^{-6} m.
1 Micro-millimetre ($\mu\mu$)	= one-millionth of a mm. = 10^{-9} m.
1 Inch	= 2·54 centimetres.
1 Foot	= 30·4797 centimetres.
1 Mile	= 1·609 kilometre.
1 Square centimetre	= 0·155 square inch.
1 Square inch	= 6·4515 square centimetres.
1 Square foot	= 929·01 square centimetres.
1 Cubic centimetre	= 0·061 cubic inch.
1 Cubic inch	= 16·3866 cubic centimetres.
1 Cubic foot	= 28316 cubic centimetres.
1 Pint	= 567·63 cubic centimetres.
1 Gallon	= 4·5435 litres.
1 Gramme	= 15·432 grains.
1 Kilogramme	= 2·205 pounds.
1 Grain	= 0·0648 gramme.
1 Ounce (avoir.)	= 28·3495 grammes.
1 Pound	= 453·59 grammes.
1 Litre	= 0·22 gallon.
1 Poundal	= 13825 dynes.
1 Foot-pound	= $1·356 \times 10^7$ ergs.
1 Foot-poundal	= $4·2139 \times 10^5$ ergs.
1 Gramme-centimetre	= 981 ergs.
1 Joule	= 10^7 ergs.
1 Horse-power	= 746×10^7 ergs per second.

1 Force-de-cheval	$= 736 \times 10^7$ ergs per second.
1 Watt	$= 10^7$ ergs per second.
1 Poundal	= 1 pound ÷ '*g.*'
1 Dyne	= 1 gramme ÷ '*g.*'
1 Megadyne	$= 10^6$ dynes = 2·247 lbs. (force).
1 Pound (force)	= 32·2 poundals = $4{\cdot}45 \times 10^5$ dynes.
1 Gramme „	= 981 dynes.
1 Gramme per square cm.	= 0·01422 lb. per sq. inch.
1 Pound per square foot	= 0·488 grammes per square cm.
1 Pound per square inch	= 70·31 „ „

The length of the metre given in the text = 39·37043 inches (p. 7) is according to the recent determination by Colonel Clarke. As the metric unit is taken at 0° C. and the British unit at 62° F., a metre used at the ordinary temperature of 62° F. is equivalent to 39·382 inches. A convenient approximation is 1 metre = 39⅜ inches, or 3 feet 3 inches and three-eighths of an inch; a rough approximation is 1 inch = 25 millimetres, or 1 decimetre = 4 inches.

Dr. Johnstone Stoney has pointed out (*Proc. Roy. Dublin Soc.*, 1889) that, without sensible error (less than ·001 to ·002 per cent), we may take the following numbers for the conversion of British and metrical measures into one another :—

Length.		*Weight.*	
yard	= 914·4 mm.	pound	= 453·6 grammes.
foot	= 304·8 „	ounce	= 28·35 „
inch	= 25·4 „	grain	= ·0648 „
metre	= 39·37 inches.	gramme	= 15·432 grains.

And with an error of only ·01 per cent we may take a gallon = 4544 c.c., pint = 568 c.c., fluid ounce = 28·4 c.c.

For the description of a gauge for the appreciation of ultra-visible quantities, see a suggestive paper by Dr. Stoney, in *Proc. Roy. Dublin Soc.*, 1892.

TABLE III.

Mensuration.

A = area of curve, S = surface of body, V = volume of body.

(1) Rectangle, sides a and b, $A = ab$.

(2) Square, side a, $A = a^2$.

(3) Parallelogram, sides a and b, height h, θ = angle between a and b, $A = bh = ab \sin\theta$.

(4) Triangle, sides a, b, c, height h, $s = \frac{1}{2}(a+b+c)$, $A = \frac{1}{2}bh = \frac{1}{2}bc\sin\theta = \{s(s-a)(s-b)(s-c)\}^{\frac{1}{2}}$.

(5) Circle of radius r, circumference $= 2\pi r$, $A = \pi r^2$.

Segment of circle, $A = \pi r^2 \dfrac{\theta}{180} - \dfrac{r^2}{2}\sin\theta$.

(6) Circular ring, r and r_1 the external and internal radius, $A = \pi(r^2 - r_1^2)$.

(7) Parabola, a = abscissa, b = double ordinate, $A = \dfrac{2}{3}ab$.

(8) Ellipse, a and b semi-axes, $A = \pi ab$.

(9) Rectangular parallelopiped of sides a, b, c, $V = abc$, $S = 2(ab + ac + bc)$.

(10) Cube, side a, $V = a^3$, $S = 6a^2$.

(11) Prism and cylinder, radius r, height h, $V = \pi r^2 h$, $S = 2\pi r(h + r)$.

(12) Pyramid and cone, radius r, height h, slant height l, $V = \frac{1}{3}\pi r^2 h$, $S = \pi r(l + r)$.

(13) Frustum of cone or pyramid, r and r_1 the radii of the ends, $V = \dfrac{\pi h}{3}(r^2 + r_1^2 + rr_1)$, $S = \pi\{r^2 + r_1^2 + l(r + r_1)\}$.

(14) Sphere of radius r, $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$.

(15) Segment of sphere, r_1 = radius of base, h = height of segment, $V = \dfrac{\pi h}{6}(h^2 + 3r_1^2) = \dfrac{\pi h^2}{3}(3r - h)$, $S = 2\pi rh + \pi r_1^2$.

TABLE IV.

Acceleration due to Gravity.

(Everett.)

Place.	Latitude.	Value of g in cms. per sec. per sec.	Length of Seconds Pendulum in cms.
Equator . .	0° 0′	978·10	99·103
Latitude 45° .	45° 0′	980·61	99·356
Paris . . .	48° 50′	980·94	99·390
Greenwich . .	51° 29′	981·17	99·413
Berlin . . .	52° 30′	981·25	99·422
Dublin . .	53° 21′	981·32	99·429
Belfast . .	54° 36′	981·43	99·440
Aberdeen . .	57° 9′	981·64	99·461
Pole . . .	90° 0′	983·11	99·610

TABLE V.

Densities.

LIQUIDS AT 15° C.			
Substance.	ρ.	Substance.	ρ.
Sea water . .	1·026	Bisulphide of carbon	1·27
Alcohol . .	0·806	Ether . . .	0·72
Glycerine . .	1·26	Chloroform . .	1·49
Mercury . .	13·56	Methylated spirit	0·83
Milk . . .	1·03	Sulphuric acid .	1·85
Turpentine . .	0·87	Nitric acid . .	1·56
Petroleum . .	0·82	Hydrochloric acid	1·17
Linseed oil . .	0·94	Sol. zinc sulphate satd. . .	1·38
Olive oil . .	0·92		
Fusel oil . .	0·81	Sol. copper sulphate satd. .	1·22
Benzol . .	0·883		

TABLE V.—*Continued.*

Densities.

Solids.			
Substance.	ρ.	Substance.	ρ.
Aluminium . .	2·6	Tin . . .	7·3
Bismuth . .	9·8	Zinc . . .	7·1
Brass (wire) . .	8·5	Gold . . .	19·3
Copper ,, . .	8·8	Silver . . .	10·5
Iron ,, . .	7·7	Platinum . .	21·5
Steel ,, . .	7·72	Oak . . .	0·86
German silver .	8·5	Beech . . .	0·75
Lead . . .	11·3	Elm . . .	0·54
Nickel . .	8·6	Cork . . .	0·24
Diamond . .	3·5	Paraffin wax .	0·89
Graphite . .	2·3	Bees' wax . .	0·96
Gas carbon . .	1·9	Phosphorus . .	1·84
Wood charcoal .	1·6	Sulphur . .	1·97
Granite . .	2·7	Sugar (cane) .	1·6
Quartz . .	2·65	Rock salt . .	2·24
Basalt . . .	2·86	Gunpowder (loose)	8·4–9·4
Slate . . .	2·5	Gutta-percha .	0·97
Porcelain . .	2·4	India-rubber .	0·99
Brick . . .	2·1	Ivory . .	1·92
Sand . . .	1·5	Marble . .	2·8
Crown glass .	2·6	Fluorspar . .	3·2
Flint glass . .	2·93 to 3·5	Calcspar . .	2·7
		Ice . . .	0·92

Note.—The student must bear in mind that, independently of their temperature, the densities of many substances vary within a considerable range, owing to their degree of purity and (in the case of solids) to their physical state. A useful table of densities is given in the *Ency. Brit.*, Art. "Hydrometer."

Table VI.

Density of Dry Air compared with Water at 4° C.

(Kohlrausch.)

Temperature.	Pressure.		
Centigrade.	750 mms.	760 mms.	770 mms.
0	0·001276	0·001293	0·001310
1	0·001271	0·001288	0·001305
2	0·001267	0·001283	0·001300
3	0·001262	0·001279	0·001296
4	0·001257	0·001274	0·001291
5	0·001253	0·001270	0·001286
6	0·001248	0·001265	0·001282
7	0·001244	0·001260	0·001277
8	0·001239	0·001256	0·001272
9	0·001235	0·001251	0·001268
10	0·001231	0·001247	0·001263
11	0·001226	0·001243	0·001259
12	0·001222	0·001238	0·001255
13	0·001218	0·001234	0·001250
14	0·001214	0·001230	0·001246
15	0·001209	0·001225	0·001242
16	0·001205	0·001221	0·001237
17	0·001201	0·001217	0·001233
18	0·001197	0·001213	0·001229
19	0·001193	0·001209	0·001224
20	0·001189	0·001204	0·001220

Table VII.

Density of Gases at Temperature 0° C. and 760 mm. Pressure.

Name.	Weight of 1000 c.c. in Grammes.	Density Compared with Air.
Air	1·2932	1·0000
Oxygen . . .	1·4298	1·1056
Hydrogen . . .	0·0896	0·0693
Nitrogen . . .	1·2562	0·9714
Coal gas (variable) .	0·6666	0·5156
Marsh gas . . .	0·7270	0·5590
Carbon monoxide .	1·2344	0·9569
Carbon dioxide . .	1·9774	1·5290
Sulphur dioxide . .	2·7289	2·1930
Chlorine . . .	3·1328	2·4216
Aqueous vapour . .	...	0·6230

The reciprocals of the weight in grammes of one c.c. of the different gases give the *volume* in c.c. of one gramme at the standard temperature and pressure. At this pressure and at a temperature of 21°·2 C. the weight of a litre (1000 c.c) of air is exactly 1·2 gramme, or 1 gramme of hydrogen, 16 grms. of oxygen, and 14 grms. of nitrogen, at this temperature and pressure, occupy 12 litres. From this Dr. Johnstone Stoney (*Proc. Roy. Dublin Soc.*, 1889) has deduced the following convenient formula for laboratory use :—

At 760 mm. pressure and say 21° C. temperature,

The weight of a litre of any gas $= D/12$ grammes,
The volume of a gramme of the gas $= 12/D$ litres,

where D is the density of the gas compared with hydrogen as unity, thus for air $D = 14·4$. At any other temperature and pressure, not far removed from the above, add or subtract 1 per cent for every 3° C. above or below 21° C., and 1 per cent for every $7\frac{1}{2}$ mm. ($\frac{3}{10}$ of an inch) above or below the standard pressure.

TABLE VIII.

Volume and Density of Water.

(Everett's *Physical Constants.*)

Temperature Centigrade.	Volume at 4° = 1.	Absolute Density. Grammes per c.c.
0	1·000129	·999884
1	1·000072	·999941
2	1·000031	·999982
3	1·000009	1·000004
4	1·000000	1·000013
5	1·000010	1·000003
6	1·000030	·999983
7	1·000067	·999946
8	1·000114	·999899
9	1·000176	·999837
10	1·000253	·999760
11	1·000345	·999668
12	1·000451	·999562
13	1·000570	·999443
14	1·000701	·999312
15	1·000841	·999173
20	1·001744	·998272
25	1·002888	·997133
30	1·004253	·995778
40	1·00770	·99236
50	1·01195	·98821
60	1·01691	·98339
70	1·02256	·97795
80	1·02887	·97195
90	1·03567	96557
100	1·04312	·95866

A collation of the best determinations of the variation of density of water with temperature will be found in a paper by Mendeléeff in the *Phil. Mag.*, Jan. 1892.

Table IX.

Volume and Density of Mercury.

(Lupton's Tables.)

Temperature Centigrade.	Volume at 0°=1.	Density. Grammes per c.c.
0	1·000000	13·596
4	1·000716	13·586
5	1·000896	13·584
10	1·001792	13·572
15	1·002691	13·559
20	1·003590	13·547
30	1·005393	13·523
40	1·007201	13·499
50	1·009013	13·474
60	1·010831	13·450
70	1·012655	13·426
80	1·014482	13·401
90	1·016315	13·377
100	1·018153	13·353

Table X.

Depression of Barometric Column due to Capillarity.

(Pouillet.)

Internal Diameter of Tube in mm.	Depression in mm.	Internal Diameter of Tube in mm.	Depression in mm.
2	4·58	12	·26
3	2·90	13	·20
4	2·05	14	·16
5	1·51	15	·13
6	1·14	16	·10
7	·88	17	·08
8	·68	18	·06
9	·53	19	·05
10	·42	20	·04
11	·33	21	·03

TABLE XI.

Elasticity and Tenacity.

(Chiefly from Sir W. Thomson's *Mathematical and Physical Papers*, vol. iii.)

Substance.	Young's Modulus. In Dynes per square cm.	Tenacity. In Grammes per square cm.
Iron (wire)	$1{\cdot}826 \times 10^{12}$	$6{\cdot}5 \times 10^{6}$
*Iron (wire, soft)	1·27 ,,	
Steel ,,	1·846 ,,	9·9 ,,
,, pianoforte wire	2·01 ,,	18·5 to 23·6 ,,
*Manganese steel (hard wire)	1·648 ,,	17·3 ,,
* ,, ,, (soft ,,)	1·47 ,,	7·7 ,,
Steel (bar)	2·0 ,,	7· to 9· ,,
Brass (cast)	0·63 ,,	1·27 ,,
,, (wire)	0·982 ,,	3·43 ,,
Copper ,, annealed	1·031 ,,	3·20 ,,
,, ,, (hard)	1·172 ,,	4·22 ,,
Lead (sheet)	0·050 ,,	0·23 ,,
,, (cast)	0·187 ,,	0·22 ,,
Tin ,,	0·409 ,,	0·42 ,,
Zinc (drawn)	0·856 ,,	1·58 ,,
*German silver (wire)	1·354 ,,	6·0 ,,
Platinum ,,	1·569 ,,	3·50 ,,
Silver (drawn)	0·722 ,,	2·96 ,,
Gold ,,	0·829 ,,	2·75 ,,
Bronze ,,	0·683 ,,	2·52 ,,
Glass	0·551 ,,	0·66 ,,
Slate	0·996 ,,	0·73 ,,
Oak	0·101 ,,	1·05 ,,
Ash	0·111 ,,	1·20 ,,
Teak	0·166 ,,	1·05 ,,

* These are our own determinations.—W. F. B.

Table XII.

Limits of Elasticity and Breaking Stress by Stretching.

(Wertheim.)

Giving the weight (p) required to stretch a wire 1 square mm. in section, so as to produce a permanent elongation of ·05 mm. per metre, *i.e.* $\frac{1}{200}$ per cent; also the breaking weight (P) for the same wire at a temperature of 15° C.

Name of Substance.	Condition of Substance.	p Grammes.	P Grammes.
Lead . .	Drawn . .	250	2,070
	Annealed .	200	1,800
Tin . . .	Drawn . .	400	2,450
	Annealed .	200	1,700
Gold . .	Drawn . .	13,500	27,200
	Annealed .	3,000	10,080
Silver . .	Drawn . .	11,000	29,000
	Annealed .	2,500	16,020
Copper . .	Drawn . .	12,000	40,300
	Annealed .	3,000	30,540
Platinum .	Drawn . .	26,000	34,100
	Annealed .	14,500	23,500
Iron . .	Drawn . .	32,500	61,100
	Annealed .	5,000	46,880
Cast steel .	Drawn . .	55,600	83,800
	Annealed .	5,000	65,700

Table XIII.

Torsional and Longitudinal Resiliences in cms.

(From Sir William Thomson's *Mathematical and Physical Papers*, vol. iii.)

Substance.	Torsional.	Longitudinal.
India-rubber band .		120,000
Pianoforte steel wire	130 to 1203	17,620
Platinoid ,,	271 to 1580	1,693
German silver ,,	168	514
Brass ,,	860 to 940	728
Delta metal ,,	750 to 1250	2,708
Phosphor bronze ,,	...	4,904
,, ,, ,,	...	3,545
,, ,, ,,	...	1,842
Silicium bronze ,,	...	3,166
Manganese ,, ,,	...	2,998

The torsional resiliences given in the above table show great differences for the same metal due to differences of temper. In like manner various specimens of *brass* show the following differences in elasticity :—

Young's modulus . .	9·48 to 10·44	$\times 10^{11}$
Simple rigidity . .	3·53 to 3·90	$\times 10^{11}$
Volume elasticity . .	10·02 to 10·85	$\times 10^{11}$

The volume elasticity (p. 172) and compressibility of *water* at different temperatures are as follows :—

	0°	11°	18°	43° C.	
Volume elasticity .	2·02	2·11	2·20	2·29	$\times 10^{10}$
Compression for one megadyne per sq. cm. . . .	4·96	4·73	4·55	4·36	$\times 10^{-5}$

Table XIV.

Moments of Inertia.

M = mass of the body,
l = length,
a, b, c = length of the sides, if the body is of rectangular cross-section,
R = external radius,
r = internal radius.

Body.	Moment of Inertia.	Direction of the Axis of Oscillation.
Uniform thin rod .	$M\frac{l^2}{3}$	At end, perpendicular to length.
,, ,, .	$M\frac{l^2}{12}$	Through middle, perpendicular to length.
Rectangular lamina .	$M\frac{a^2}{12}$	Through centre, *parallel* to one side, and bisecting side a.
,, ,, .	$M\frac{a^2+b^2}{12}$	Through centre, perpendicular to plane.
Rectangular parallelopiped . . .	$M\frac{a^2+b^2}{12}$	Perpendicular to side contained by a and b.
Circular plate . .	$M\frac{R^2}{2}$	Through centre, perpendicular to plane of plate.
,, ring . .	$M\frac{R^2+r^2}{4}$	Through centre, perpendicular to plane of ring.
Right cylinder . .	$M\frac{R^2}{2}$	The axis of figure.
,, ,, . .	$M\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$	Through centre, perpendicular to axis of cylinder.
Hollow cylinder .	$M\frac{R^2+r^2}{2}$	Axis of figure.
,, ,, .	$M\left(\frac{l^2}{12}+\frac{R^2+r^2}{4}\right)$	Through centre, perpendicular to axis of cylinder.
Sphere . . .	$\frac{2}{5}MR^2$	Any diameter.

Table XV.

Rigidity Moduli.

(In grammes per square cm.)

Substance.	n.	Substance.	n.
Brass . .	350×10^6	Copper . .	456×10^6
Iron (wrought)	785 ,,	German silver	496 ,,
Iron (cast) .	542 ,,	Platinoid .	476 ,,
Steel . .	834 ,,	Glass . .	241 ,,

Table XVI.

Scale of Hardness.

H = 1. Talc (common laminated variety).
H = 2. Gypsum (crystallised).
H = 3. Calcspar (transparent).
H = 4. Fluorspar (crystalline).
H = 5. Apatite (transparent).
H = 6. Felspar (cleavable).
H = 7. Quartz (transparent).
H = 8. Topaz (transparent).
H = 9. Sapphire (cleavable).
H = 10. Diamond.

Bodies scratched by finger nail .	H = 2·5 or less.
Bodies that scratch copper .	H = 3 or more.
Polished white iron . . .	H = 4·5.
Window glass . . .	H = 5 to 5·5.
Bodies scratched by a penknife .	H = less than 6.
Steel point or file . . .	H = 6 to 7.
Flint	H = 7.

Table XVII.

Coefficients and Angles of Friction: without lubricants (Morin).

Substance and position fibres.	On Point of Motion.		In Motion.	
	ϕ.	μ.	ϕ.	μ.
Oak on oak (parallel) .	31° 50′	0·62	25° 40′	0·48
,, ,, (crosswise)	28° 20′	0·54	18° 45′	0·34
,, ,, (endwise) .	23° 20′	0·43	10° 45′	0·19
Elm on oak (parallel) .	34° 40′	0·69	23° 20′	0·43
,, ,, (crosswise)	29° 40′	0·57	24° 15′	0·45
Wrought iron on oak (parallel) . .	31° 50′	0·62	31° 50′	0·62
Copper on oak (parallel)	31° 50′	0·62	31° 50′	0·62
Wrought iron on cast iron . . .	10° 45′	0·19	10° 10′	0·18
Cast iron on cast iron	9° 5′	0·16	8° 30′	0·15

Table XVIII.

Compressibility (Quincke).

Substance.	Compression in Millionths for one Atmosphere.		Temp. $t°$ C.
	At 0° C.	At $t°$ C.	
Mercury	2·95	1·87	15·00
Glycerine	25·24	25·10	19·00
Rape oil	48·02	58·18	17·80
Almond oil	48·21	56·30	19·68
Olive oil	48·59	61·74	18·30
Water (see also p. 275) .	50·30	45·63	22·93
Bisulphide of carbon .	53·92	63·78	17·00
Oil of turpentine . .	58·17	77·93	18·56
Benzol	...	62·84	16·08
Alcohol	82·82	95·95	17·51
Ether	115·57	147·72	21·36
Petroleum	64·99	74·50	19·23

TABLE XIX.

Viscosity of Liquids.

Substance.	Temperature cent.	Coefficient of Viscosity.
Water	0°	0·01783
,,	10°	0·01309
,,	20°	0·0102
,,	50°	0·0056
,,	80°	0·0036
Water with		
94·00 % glycerine	8·5°	7·444
80·31 ,,	,,	1·022
64·05 ,,	,,	0·222
49·75 ,,	,,	0·093
Alcohol meth.	10°	0·0069
,, ethyl.	,,	0·0154
Glycerine	2·8°	42·180
,,	26·5°	4·944
Mercury	17·2°	0·016

TABLE XX.

Viscosity of Gases (between 15° *and* 20° *C.).* From Graham's Experiments.

Gas.	Coefficiency of Viscosity.	Transpiration Time, Oxygen = 1.
Air	0·000192	0·901
Oxygen	0·000212	1·000
Hydrogen	0·000093	0·437
Nitrogen	0·000184	0·875
Chlorine	0·000141	0·666
Marsh gas	0·000120	0·551
Olefiant gas	0·000109	0·507
Carbon monoxide	0·000184	0·875
Carbon dioxide	0·000160	0·727
Air at 0° C., *recent observations*	0·000168	

Table XXI.

Effusion and Diffusion of Gases (Graham).

Air = 1.

Gas.	Density.	$\frac{1}{\sqrt{\text{density}}}$.	Rate of Effusion.	Diffusion Volume.
Hydrogen .	0·0693	3·7998	3·613	3·83
Marsh gas .	0·5590	1·3375	1·322	1·34
Carbonic oxide .	0·9678	1·0165	1·012	1·01
Nitrogen . .	0·9713	1·0147	1·016	1·01
Oxygen . .	1·1056	0·9510	0·950	0·95
Nitrous oxide .	1·5270	0·8092	0·834	0·82
Carbonic dioxide	1·5290	0·8087	0·821	0·81

For table of coefficients of inter-diffusion of gases in sq. cms. per sec., see Maxwell's *Heat*, last ed., p. 342.

Table XXII.

Surface Tensions at 20° C. in grammes and dynes per Lineal Centimetre (Quincke).

Substance. Surface in contact with air.	Grammes.	Dynes.	Surface Tension Water=1.
Water	0·0826	81·0	1·000
Mercury	0·5504	540·0	6·667
Bisulphide of carbon . . .	0·0327	32·1	0·396
Chloroform	0·0312	30·6	0·378
Alcohol	0·0260	25·5	0·315
Olive oil	0·0376	36·9	0·455
Turpentine	0·0303	29·7	0·367
Petroleum	0·0323	31·7	0·391
Hydrochloric acid	0·0715	70·1	0·865
Solution of hyposulphate of soda .	0·0790	77·5	0·957

Table XXIII.

Surface Tensions (Various observers).

The values found by Quincke for water and mercury given in the last table are only applicable when special precautions are taken to avoid impurity and surface contamination. In fact, "It seems doubtful whether the tension of water is really so high as that recorded by Quincke. Observations upon very clean surfaces, in which the tension was determined from its effect upon the propagation of ripples, gave 0·074 grammes" = 72·6 dynes per cm. (Rayleigh). The following table gives the values of the surface tension, T, of different liquids found by other observers.

Substance.	Density.	T. in dynes.	Temp.	Authority.
Water	1·00	75·2	0° C.	Brunner.
,, ...	,,	74·2	8	Desains.
Mercury	13·56	453·2	15	,,
Sulphuric acid	1·85	62·1	14	Frankheim.
Nitric acid	1·50	41·9	16	,,
Absolute alcohol . . .	0·79	22·3	20	Wilhelmy.
Amylic ,, . . .	0·82	23·8	,,	,,
Ether	0·72	17·6	,,	Mendeléeff.
Solution of soap 1 to 40 . .	1·01	27·7		Mensbrugghe
Solution of saponine . .		45·8		,,
Water containing camphor .		44·2		,,

In connection with the behaviour and measurement of contaminated water surfaces, Miss Pockels' recent valuable and suggestive experiments on surface tension should be read; see *Nature*, vol. xliii. p. 437, and vol. xlvi. p. 418.

REDUCTION OF RESULTS—PROBABLE ERROR

(i.) After a series of observations have been made in an experiment, numerical reductions have generally to be made before the final result can be obtained. Before beginning these numerical reductions the observations should be arranged in a tabular form. Tables of logarithms and other tables should be used when necessary to lessen the labour of calculation. Whilst for most physical calculations four figure logarithms are sufficient, the student should be on his guard against trusting to these when accuracy is required in certain kinds of calculations, such as correcting the density of water for temperature where the first significant figure may occur in the fifth or sixth decimal place. The student ought, however, to avoid the tendency to over-refinement in his calculations, such as using many decimal places when the experiments do not warrant it, a good general rule is to go one decimal place beyond that which denotes the accuracy which he expects from his result.

(ii.) The degree of accuracy attainable in an experiment varies very much with the precision of the instrument employed; the errors due to the instrument alone are called *constant errors*, and it is very important that they should be found out and allowed for in the final result. Thus it would be useless to read the barometer to the hundredth of an inch if the barometer itself be unreliable, either through the Torricellian vacuum being imperfect or the instrument not quite vertical, or to read the cathetometer to a tenth of a millimeter if the scale be badly divided or the levelling imperfect. We should always, in the first place, ascertain what reliance can be placed in the instrument we are using either by comparison with a standard or other means. It is very desirable that the student should if possible *check his*

results by another method of experiment which, even if less sensitive, will serve to reveal gross errors. Thus in finding the internal diameter of a capillary tube by the method of weighing a given length of mercury, the result may be checked by the diameter of the tube being measured directly by means of the reading microscope.

(iii.) If in an experiment two instruments are employed which possess different degrees of accuracy, it would be useless to read or observe to the extreme degree of accuracy of the most delicate one. Thus in an experiment involving the product of a weight into a temperature it would be waste of time and give a misleading sense of accuracy to weigh to a milligramme if the thermometer only read to half a degree.

(iv.) It is also desirable to make each experiment under conditions which give the most favourable result. Thus in determining the specific gravity of a body, it is best to use as large masses as the balance will weigh, for although the sensitiveness of a balance is less with a heavy load, the loss in this respect is more than compensated by the gain derived owing to the smaller percentage error (see p. 38).

(v.) When all the separate determinations of an experiment are entitled to an equal degree of confidence, the *arithmetic mean* of the whole gives the most probable value of the required result. If the separate determinations be now compared with the mean value, differences greater or less will be found to exist, and from these differences the probable error of a single observation, as well as the probable error of the result, may be deduced as follows:—

If $n =$ the number of observations,

$\delta_1, \delta_2, \delta_3$ etc. $=$ the differences,

$S = \delta_1^2 + \delta_2^2 + \ldots \delta_n^2$ or the sum of their squares,

then the *probable error of a single observation* will be

$\pm 0{\cdot}6745\sqrt{\dfrac{S}{n-1}}$. And the *probable error of the result* is $\pm 0{\cdot}6745\sqrt{\dfrac{S}{n(n-1)}}$.*

(vi.) The graphic method (Expt. 15, p. 40) affords another means of detecting errors in a series of observations. By plotting the results on millimetre paper, and using a flexible rule to draw a smooth curve, small irregularities are at once seen, which indicate errors of experiment. Care must, however, be taken not to carry the smoothing process too far. The results being shown in a curve, the intervening values not actually determined by experiment can now readily be ascertained by mere inspection; in this way "interpolation" is best accomplished, otherwise recourse must be had to the more troublesome interpolation formulæ.

* See Airy's *Theory of Errors of Observation*, part 1, p. 24. A full discussion of the subject of mean and probable error will be found in the introduction of Kohlrausch's *Physical Measurements*.

THE END

Printed by R. & R. Clark, *Edinburgh*

Practical Physics. An Introductory Handbook for the Physical Laboratory. In Three Parts. By W. F. BARRETT and W. BROWN. Part I., Physical Processes and Measurements—The Properties of Matter—Heat. Part II., Sound and Light. Part III., Electricity and Magnetism—Electrical Measurements.

English Classics for Schools.

General Editors—
THE REV. M. G. GLAZEBROOK, M.A.,
And C. E. VAUGHAN, M.A.

Grade I., for the ages of 10 to 13.
Each Volume 6d. net.

English Ballads (FIRST SERIES).
English Ballads (SECOND SERIES).
Macaulay's Essay on Clive.*
Macaulay's Essay on Warren Hastings.*
Scott's Ivanhoe.* 2 Vols.
Scott's Kenilworth.*
Scott's Lady of the Lake.* 3 Vols.
Wordsworth's Shorter Poems.

Grade II., for the ages of 13 to 16.
Each Volume 10d. net.

Cook's Voyages.*
Defoe's Plague of London.*
Longfellow's Evangeline.
Pope's Iliad.* 3 Vols.
Prescott's Conquest of Mexico.* 2 Vols.

Grade III., for the ages of 16 to 18.
Each Volume 1s. 4d. net.

Burke on French Revolution.*
Selection from Clarendon's History of the Rebellion.
Selection from Lamb's Essays.
More's Utopia.*

* The text of these books is reduced.

An Elementary History of England. For the use of the Lower Forms of Schools. By CYRIL RANSOME. 1s. 6d. net.

An Advanced History of England. For use in Colleges and Upper Forms of Schools. By CYRIL RANSOME.

A History of Scotland. For Junior Classes. By L. W. LYDE. 1s. 6d.

Outlines of Roman History. By H. F. PELHAM.

An Elementary Geography of British North America. By L. W. LYDE.

Outlines of Geography. For the use of Lower and Middle Forms of Schools, and of Candidates for the Army Preliminary Examinations. By A. A. SOMERVILLE, and R. W. WHITE THOMSON. 3s. 6d.

Miscellaneous Essays. By G. SAINTSBURY. 7s. 6d.

Essays in English Literature. 1780 to 1860. By G. SAINTSBURY. 7s. 6d.

The History of English. By A. C. CHAMPNEYS.

The Moral Influence of Literature, and Classical Education in the Past and at Present. By HENRY NETTLESHIP. 1s. 6d.

English Literature Teaching in Schools. By H. COURTHOPE BOWEN. 1s. 6d.

Outlines of Logic, Psychology, and Ethics. By A. BAKER. 4s. 6d.

The Structure of English Sentences. An Aid to Translation and Composition. By R. SOMERVELL. 1s. 6d.

Macaulay's Lays of Ancient Rome. By H. T. RHOADES. 1s. 6d.

Ballad and other Poetry. By H. R. HEATLEY. 1s. 4d.

Lessons from the Old Testament. Selected and Arranged by the Rev. M. G. GLAZEBROOK.

Senior Course. In Two Volumes. Vol. I., The Creation to the Death of Saul. Vol. II., The Death of Saul to Nehemiah. Cloth limp, 2s. 6d. each volume; or in cloth boards, 3s. each volume.

Junior Course. In One Volume. The Creation to Nehemiah. Cloth limp, 2s. 6d.; or cloth boards, 3s.
Or, in Three Parts, cloth limp, 1s. each. Part I., The Creation to the Settlement of the Tribes. Part II., Othniel, Ehud, and Barak, to Rehoboam and Jeroboam. Part III., Rehoboam and Jeroboam to Nehemiah.